Math 135 Final Exam Review Session

1) Solve the following system of linear congruences:

$$11x \equiv 12 \pmod{24}$$
$$x \equiv 4 \pmod{25}$$

Solution By definition x = 4 + 25k for some $k \in \mathbb{Z}$. Plug into the first equation gives:

$$11(4+25k) \equiv 12 \pmod{24}$$

Simplifying gives:

 $44 + 275k \equiv 12 \pmod{24}$ $20 + 35k \equiv 12 \pmod{24}$ $11k \equiv 16 \pmod{24}$

Since $5 \cdot 24 = 120$, multiplying both sides by 11 yields:

$$11 \cdot 11k \equiv 11 \cdot 16 \pmod{24}$$
$$121k \equiv 176 \pmod{24}$$
$$k \equiv 8 \pmod{24}$$

Therefore $k = 8 + 24\ell$ for some $\ell \in \mathbb{Z}$.

Thus, $x = 4 + 25k = 4 + 25(8 + 24\ell) = 204 + 600\ell$. Thus $x \equiv 204 \pmod{600}$