~~Check Page order.~~

Math 135
Live stream

Starts: 6:00pm

~10 second delay.

Stream is recorded!

Suppose S and T are sets.
Show if $S \cap T = S$ then $\underbrace{S \subseteq T}$.
What about the converse?

Pf: Let $x \in S$. Then since $S \cap T = S$, we have that $x \in S \cap T$. Thus, $x \in S$ and $x \in T$.

$\therefore x \in T$, and $S \subseteq T$. If $S = \emptyset$, note $\emptyset \subseteq T$ for any set T.

// Converse: $S \subseteq T \Rightarrow S \cap T = S$.

$S \subseteq T \Rightarrow S \cap T = S$.

Pf: Claim: $x\in$ $S \cap T \subseteq S$. ✓

$S = \{1,2,3\}$

$T = \{1,2,3,4,5,6\}$

Claim: $S \subseteq S \cap T$. #True if $S = \emptyset$.

Let $x \in S$. Then $x \in T$ since $S \subseteq T$. Therefore, $x \in S \cap T$. ∎.

$\therefore$ Since $S \cap T \subseteq S$ and $S \subseteq S \cap T$, $S = S \cap T$.

In a strange country, there are 4 and 5 cent coins. Prove that any integer amount of currency greater than 12 cents can be formed

$13 = 5 + 4 + 4$ $\quad$ $15 = 5+5+5$ $\quad$ $17 = 5+4+4+4$

$14 = 5 + 5 + 4$ $\quad$ $16 = 4+4+4+4$ $\quad$ $18 = 5+5+4+4$

$19 = 5+5+5+4$

Let $P(n)$ be the statement that n can be represented using 4 or 5 cent coins.

We want to show that $P(n)$ is true forall $n \geq 13$, $n \in \mathbb{N}$.

BC: When $n=13$, $13=5+4+4$. ✓

We also did $n=14, 15, 16$.

IH: Assume ~~P(K)~~ is true for some $K \in \mathbb{N}$, ~~K≥13~~
$P(13), P(14), P(15), \ldots P(K)$ $K \geq 16$

IC: For $P(K+1)$, I want that $(K+1)-4$ is a sum of 5s and 4s. Note $K+1-4 = \underline{K-3 \leq K}$. ($K-3 \geq$

So $P(K-3)$ is true by IH. Therefore $K-3$ is a sum of 5s and 4s. Hence $(K-3)+4 = K+1$ is also a sum of 5s and 4s. $\therefore P(K+1)$ is true $\therefore P(n)$ is true $\forall n \geq 1$ by POS

Always True.

Show $\overbrace{((P \Rightarrow Q) \Rightarrow R) \vee (\neg P \vee Q)}^{S}$ is a tautology.

P	Q	R	$P \Rightarrow Q$	$((P \Rightarrow Q) \Rightarrow R)$	$\neg P$	$\neg P \vee Q$	S
T	T	T	T	T	F	T	T
T	T	F	T	F	F	T	T
T	F	T	F	T	F	F	T
T	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	F	T	T	T
F	F	T	T	T	T	T	T
F	F	F	T	F	T	T	T ✓

$\therefore$ S is a tautology since S is always True! $\square$.

$$((P \Rightarrow Q) \Rightarrow R) \vee (\neg P \vee Q)$$

$$\equiv ((\neg P \vee Q) \Rightarrow R) \vee (\neg P \vee Q) \quad \text{(By class)}$$

$$\equiv (S \Rightarrow R) \vee S \qquad \text{where } S = \neg P \vee Q.$$

If S is true then $(S \Rightarrow R) \vee \boxed{S}$ is true.

If S is false then $S \Rightarrow R$ is True thus $(S \Rightarrow R) \vee S$ is tr

Hence original is a tautology. ⊠.

Q: Are the following onto? 1:1?

$\rightarrow\leftarrow$.

(a) $f: \mathbb{Z} \to \mathbb{Z}$, $f(n) = 2n+1$

Recall: 1:1 $\forall x, y \in$ Domain $f(x) = f(y) \Rightarrow x = y$.

onto $\forall y \in$ Codomain $\exists x \in$ Domain $f(x) = y$.

Assume $m, n \in \mathbb{Z}$ are s.t. $f(m) = f(n)$. Then

$$2m + \cancel{1} = 2n + \cancel{1}$$

$$\boxed{x = \frac{y-1}{2}}$$

$$\cancel{2}m = \cancel{2}n$$

$$m = n \qquad \therefore f \text{ is } 1:1.$$

$f(1) = 2(1)+1 = 3$ $\quad f(2) = 2(2)+1 = 5$ $\quad f(3) = 2(3)+1 = 7$.

Claim: f is NOT onto. Pf: BWOC suppose $\exists x \in \mathbb{Z}$ s.t.

$f(x) = 2$. Then $2x+1 = 2 \Rightarrow 2x = 1 \Rightarrow x = \frac{1}{2}$ # $\therefore$ f is NOT onto.

(c) $f: (\mathbb{R}-\{2\}) \to (\mathbb{R}-\{5\})$ by $f(x) = \frac{5x+1}{x-2}$.

$\boxed{1:1}$ Let $x, y \in \mathbb{R}-\{2\}$ be s.t. $f(x)=f(y)$. Then

$$\frac{5x+1}{x-2} = \frac{5y+1}{y-2}$$

$$(5x+1)(y-2) = (5y+1)(x-2)$$

$$\cancel{5xy} - 10x + y - \cancel{2} = \cancel{5xy} - 10y + x - \cancel{2}$$

$$11y = 11x \Rightarrow x = y \quad ✓. \quad (\therefore 1:1)$$

Onto. $f(1) = \frac{5+1}{1-2} = \frac{6}{-1} = -6 \quad f(\pi) = \frac{5\pi+1}{\pi-2}$.

Guess: This onto. Let $y \in \mathbb{R}-\{5\}$. Set $x = \frac{2y+1}{y-5}$. Since $y \neq 5$, x is well defined. Further, $x \neq 2$ ~~(If x=2 then show that y=5)~~. because if $x=2$, then $2 = \frac{2y+1}{y-5} \Rightarrow 2y-10 = 2y+1 \Rightarrow -10 = 1$ #.

$\therefore x \in \mathbb{R}-\{2\}$. Then, note

$$f(x) = f\left(\frac{2y+1}{y-5}\right) = \frac{5\left(\frac{2y+1}{y-5}\right)+1}{\frac{2y+1}{y-5}-2} = \frac{\frac{10y+\cancel{5}+y-\cancel{5}}{\cancel{y-5}}}{\frac{\cancel{2y}+1-\cancel{2y}+10}{\cancel{y-5}}} = \frac{11y}{11} = y$$

$\therefore f$ is onto. ∎.

n^2

9)d). Prove or disprove: $\exists n \in \mathbb{N}, \forall m \in \mathbb{Z}, -nm < 0$.

$\forall n \in \mathbb{N} \ \exists m \in \mathbb{Z} \ -nm \geq 0$.

Proof of negation: Let $n \in \mathbb{N}$. Choose $m = 0$. Then

$-n \cdot m = 0$ ✓ ~~Hen~~

$\therefore$ Negation is true. $\therefore$ original is false. ⧄.

$\forall m \in \mathbb{Z} \ \exists n \in \mathbb{N} \ -n \cdot m < 0$

$\forall$people $\exists$parent vs $\exists$parent $\forall$people.

there exists a unique.

Let S be a set. Prove $\exists!$ set T with $T \subseteq S$ and $S \cap T = S$.

~~Claim: $T = S$~~ Claim: $T = S$.

Existence: Note that $S \subseteq S$ and $S \cap S = S$ ✓.

Uniqueness: Suppose $\exists$ sets T_1 and T_2 s.t. $T_1 \subseteq S$ and $S \cap T_1 = S$ and $T_2 \subseteq S$ and $S \cap T_2 = S$.

Show: $T_1 = T_2$.

Claim: $T_1 \subseteq T_2$.

Pf: Let $x \in T_1$. Then since $T_1 \subseteq S$, $x \in S$. Further $x \in S \cap T_2$. Hence $x \in S$ and $x \in T_2$. $\therefore T_1 \subseteq T_2$.

Similarly, $T_2 \subseteq T_1$. $\therefore T_1 = T_2$. $\blacksquare$

Fibonacci sequence: $f_1 = 1$ $f_2 = 1$, $f_m = f_{m-1} + f_{m-2}$ $\forall m \geq 3$

(a) Prove for $n \geq 2$, $f_1 + f_2 + \cdots + f_{n-1} = f_{n+1} - 1$.

Pf: By induction. Let $P(n)$ be the statement that

$$f_1 + f_2 + \cdots + f_{n-1} = f_{n+1} - 1.$$

BC: When $n=2$, LHS $= f_1$ RHS $= f_{2+1} - 1 = f_3 - 1 = 2 - 1 = 1 = f_1$ ✓

IH: Assume $P(k)$ is true for some $k \in \mathbb{N}$, $k \geq 2$

$$f_1 + \cdots + f_{k-1} = f_{k+1} - 1.$$

IC: For $k+1$, WANT $f_1 + f_2 + \cdots + f_{(k+1)-1} = f_{(k+1)+1} - 1$.

$$\begin{aligned} f_1 + f_2 + \cdots + f_k &= (f_1 + f_2 + \cdots + f_{k-1}) + f_k \\ &\overset{IH}{=} f_{k+1} - 1 + f_k \\ &= (f_k + f_{k+1}) - 1 \\ &= f_{k+2} - 1 \end{aligned}$$

By def'n valid ✓ $\because k+2 \geq 3$.

$\therefore P(k+1)$ is true. $\therefore P(n)$ is true $\forall n \geq 2$ by POMI ⊡.