Prove directly from the definition that $n + \log n = O(n \log n)$.

Recall: $g(n) = O(f(n))$ if and only if there exists a real positive constant $C$ and a real constant $N$ such that for all $n \geq N$, we have that $|g(n)| \leq C |f(n)|$.

**Proof:** Choose $N = 4^{2^3}$. Then, for all $n \geq N$, we have

$$n + \log n \leq n \log n + n \log n$$

which is true since $4^{2^3} \geq 3$ and the $\log n$ is true. Thus $n \leq n \log n$ and $\log n \leq n \log n$. This is true since $n \log n \leq 2n \log n = 2 \log n$. Therefore, $n + \log n \leq 2n \log n = 2 \log n$. So setting $C = 2$, by the above definition, we have that

$n + \log n \in O(n \log n)$.