Prove directly from the definition that $n + \log n = O(n \log n)$.

Recall: $g(n) = O(f(n))$ if and only if there exists a real positive constant C and a real constant N such that for all $n \geq N$, we have that $|g(n)| \leq C|f(n)|$.

Pf: Choose $N = 3$. Then, for all $n \geq N$, we have

$$n + \log n \leq n \log n + n \log n$$

which is true since $n \geq 3$ and then $\log n \geq 1$ is true and thus $n \leq n \log n$ and $\log n \leq n \log n$. Thus,

$$|n + \log n| \overset{\text{since } n \geq 1}{=} n + \log n \leq 2n \log n \overset{\text{since } n \geq 1}{=} 2|n \log n|$$

So setting $C = 2$, by the above definition, we have that

$$n + \log n \in O(n \log n)$$