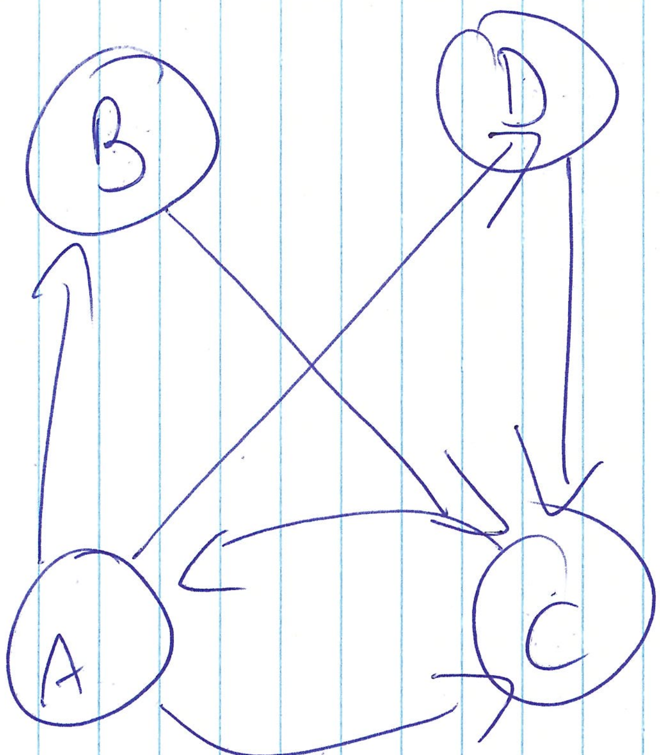
1



$r(A)+r(B)+r(C)+r(D)=1$. (1)

$r(A)=r(C)$ (2)

$r(B)=r(A)/3 = r(C)/3$ (3)

$r(C)=\frac{r(A)}{3}+r(B)+r(D)$

$r(D)=\frac{r(A)}{3}=\frac{r(C)}{3}$ (4)

$$r(P)=\sum_{Q_i \to P}\frac{r(Q_i)}{|Q_i|}$$

$|Q_i|$: # of outlinks.

$$\boxed{r(P)=\frac{1-\delta}{N}+\delta\sum_{Q_i \to P}\frac{r(Q_i)}{|Q_i|}}$$

Use eqn (1):

$r(C)+\frac{r(C)}{3}+r(C)+\frac{r(C)}{3}=1$.

$\frac{8}{3}r(C)=1$

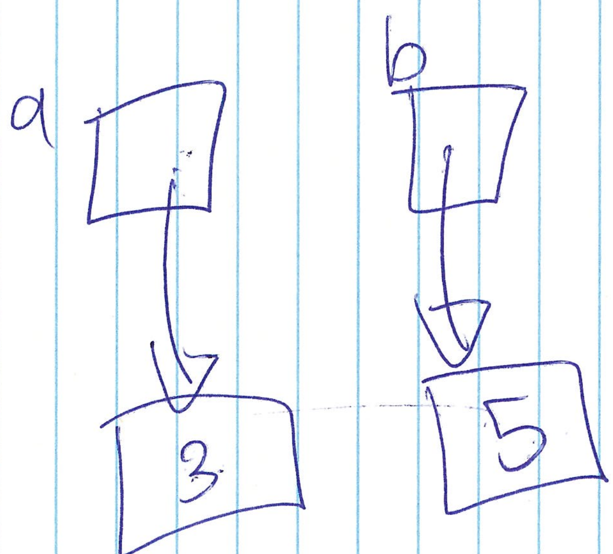
$r(C)=\frac{3}{8}$ (5)

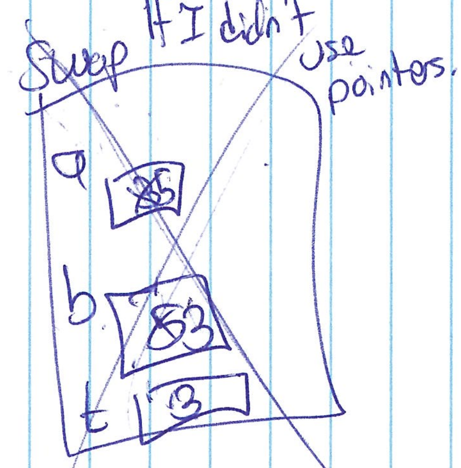
(5) into (2) gives $r(A)=\frac{3}{8}$. (5) into (3) and (4) gives $r(B)=r(D)=r(C)/3=\frac{1}{8}$.

Quick Sort.

②

Last thing, swap a[0] and a[m].





4

Clicker

Recalling our table below, how many ones in UTF-8 would have been used for our encoding of the arrow U+2B3F?

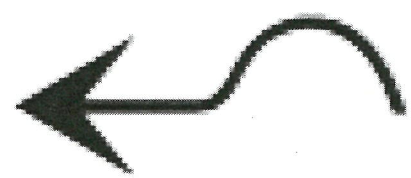
Code Point Range in Hex	UTF-8 Byte Sequence in Binary
000000-00007F	0xxxxxxx
000080-0007FF	110xxxxx 10xxxxxx
000800-00FFFF	1110xxxx 10xxxxxx 10xxxxxx
010000-10FFFF	11110xxx 10xxxxxx 10xxxxxx 10xxxxxx

7 bits
11 bits
16 bits
21 bits

2B3F = 0010 1011 0011 1111

7FF 0111 1111 1111

11 ones



a) 11

b) 12

c) 13

d) 14

e) None of the above

(5)

```
void fcn1(int n){
  int s=0;
  for(int i=0; i<n; i++){
    for(int j=0; j<i; j++){
      s++;
    }
  }
}
```

$$\text{Runtime} = \sum_{i=0}^{n-1} \sum_{j=0}^{i-1} C \quad \text{(for some constant } C\text{)}$$

$$= C \sum_{i=0}^{n-1} \sum_{j=0}^{i-1} 1$$

$$= C \sum_{i=0}^{n-1} i = C\left(\frac{(n)(n-1)}{2}\right)$$

$$= O(n^2) \qquad \boxed{\sum_{i=1}^{n} i^k = O(n^{k+1})}$$

```
void fcn2(int n){
  int s=0;                       i<√n
  for(int i=0; i*i<n; i++){
    for(int j=1; j<i; j*=4){
      for(int k=0; k<pow(3,j); k++){
        s++;
      }
    }
  }
}
```

$$\text{Runtime} = \sum_{i=0}^{\sqrt{n}} \sum_{j=1}^{\log_4 i} \sum_{k=0}^{3^j-1} C \quad \text{for some constant } C$$

$$= C \sum_{i=0}^{\sqrt{n}} \sum_{j=1}^{\log_4 i} 3^j$$

$$= C \sum_{i=0}^{\sqrt{n}} \frac{3(3^{\log_4 i}-1)}{3-1}$$

$$3^1+3^2+3^3+\dots+3^{\log_4 i}$$

$$\frac{a(r^N-1)}{r-1}$$

(6)

How many times does the "j" loop run? Ans: ℓ times where

$$4^{\ell} < i \leq 4^{\ell+1}.$$

$\Leftrightarrow$ roughly $\ell = \log_4 i$.

$$O(\log_b n) = O(\log n)$$

$$O(\log_{9973}(\log n)) = O(\log \log n).$$

$$\leq \frac{3}{2} c \sum_{i=0}^{\sqrt{n}} 3^{\log_4 i}$$

$$= \frac{3}{2} c \sum_{i=0}^{\sqrt{n}} i^{\frac{1}{\log_3 4}}.$$

$$= O\left(\sqrt{n}^{\frac{1}{\log_3 4}+1}\right).$$

$$= O\left(n^{\left(\frac{1+\log_3 4}{2\log_3 4}\right)}\right)$$

exponent:

$$\boxed{\frac{1+\log_3 4}{2\log_3 4}}.$$

$$a \log n = \log n^a$$

$$3^{\log_4 i} = 3^{\frac{\log_3 i}{\log_3 4}}$$

$$= \cancel{3^{\log_3}} i^{\frac{1}{\log_3 4}}$$

$$= i^{\frac{1}{\log_3 4}}$$

L'Hopital's rule. ⑦.

Show $n \log n \in O(n^{1.01})$

Pf. Recall if $\lim_{n\to\infty} \frac{f(n)}{g(n)}$ is finite then $f(n) = O(g(n))$.

Evaluate $\lim_{n\to\infty} \frac{n \log n}{n^{1.01}} = \lim_{n\to\infty} \frac{\log n}{n^{0.01}} \overset{LH}{=} \lim_{n\to\infty} \frac{\left(\frac{1}{n}\right)}{0.01 n^{-0.99}}$

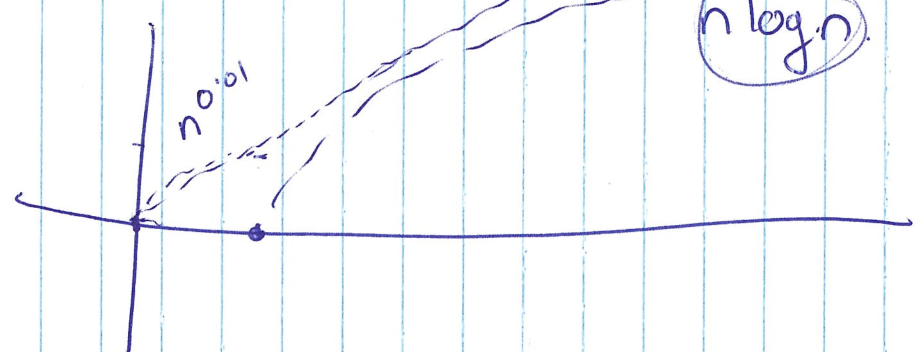
$= \lim_{n\to\infty} \frac{1}{0.01 n^{0.01}} = 0.$

$\therefore$ from class, we have that $n \log n \in O(n^{1.01})$ ☒.

$\log n$ $\quad n^{0.01}$

10^{60} $\quad 10^{0.01 \cdot 10^{60}}$

$n^{0.01}$ $\quad n \log n$ $\quad 60 \cdot 10^{60}$ $\quad n^{1.01}$



$10^{60.60}$

$60 \quad 10^{0.6}$

$10^{10^{60}}$