Week 4 List of Theorems

De Morgan's Laws (DML)
Let A and B be statements. Then

$$\neg(A \vee B) \equiv \neg A \wedge \neg B \qquad \text{and} \qquad \neg(A \wedge B) \equiv \neg A \vee \neg B$$

Bounds by Divisibility (BBD)
Let $a, b \in \mathbb{Z}$. If $a \mid b$ and $b \neq 0$, then $|a| \leq |b|$.

Transitivity of Divisibility (TD)
Let $a, b, c \in \mathbb{Z}$. If $a \mid b$ and $b \mid c$, then $a \mid c$.

Divisibility of Integer Combinations (DIC)
Let $a, b, c \in \mathbb{Z}$. If $a \mid b$ and $a \mid c$, then for all $x, y \in \mathbb{Z}$, $a \mid (bx + cy)$.

Division Algorithm (DA)
If $a \in \mathbb{Z}$ and $b \in \mathbb{N}$, then there exist unique integers q and r such that $a = qb + r$ where $0 \leq r < b$.

Well Ordering Principle (WOP) [Optional]
Every nonempty subset of the natural numbers contains a least element.

Principle of Mathematical Induction (POMI)
Let $P(n)$ be a statement. If

1. $P(1)$ is true.
2. $P(k) \Rightarrow P(k+1)$ is true for all $k \in \mathbb{N}$

then $P(n)$ is true for all $n \in \mathbb{N}$.

Principle of Strong Induction (POSI)
Let $P(n)$ be a statement. If

1. $P(1), P(2), ..., P(b)$ are true for a suitable $b \in \mathbb{N}$.
2. $P(1) \wedge P(2) \wedge \cdots \wedge P(k) \Rightarrow P(k+1)$ is true for all $k \in \mathbb{N}$

then $P(n)$ is true for all $n \in \mathbb{N}$.