Week 11 (and 12) List of Theorems

Division Algorithm for Polynomials (DAP)
If $f(x), g(x) \in \mathbb{F}[x]$ and $g(x)$ is not the zero polynomial, then there exist unique $q(x), r(x) \in \mathbb{F}[x]$ such that

$$f(x) = q(x)g(x) + r(x)$$

where $\deg(r(x)) < \deg(g(x))$ or $r(x)$ is the zero polynomial.

Remainder Theorem, (RT)
The remainder when a polynomial $f(x)$ is divided by $(x - c)$ is $f(c)$.

Factor Theorem (FT)
The linear polynomial $(x - c)$ is a factor of the polynomial $f(x)$ if and only if $f(c) = 0$.

Fundamental Theorem of Algebra (FTA)
For all complex polynomials $f(x)$ with $\deg(f(x)) \geq 1$, there exists $x_0 \in \mathbb{C}$ such that $f(x_0) = 0$.

Complex Polynomials of Degree n Have n Roots (CPN)
If $f(z)$ is a complex polynomial of degree $n \geq 1$, then $f(z)$ has n roots $c_1, c_2, \ldots, c_n \in \mathbb{C}$ and can be written as $c(z - c_1)(z - c_2) \cdots (z - c_n)$ for some $c \in \mathbb{C}$.

Rational Roots Theorem (RRT)
Let $f(x) = a_n x^n + \cdots + a_1 x + a_0$ where $a_0, \ldots, a_n \in \mathbb{Z}$, $a_n \neq 0$.
If $\frac{p}{q}$ is a root of $f(x)$ with $p, q \in \mathbb{Z}$ and $\gcd(p, q) = 1$, then $p \mid a_0$ and $q \mid a_n$.

Conjugate Roots Theorem (CJRT)
Let $f(x) \in \mathbb{R}[x]$. If $c \in \mathbb{C}$ is a root of $f(x)$, then $\overline{c}$ is also a root of $f(x)$.

Real Quadratic Factors (RQF)
Let $f(x) \in \mathbb{R}[x]$. If $c \in \mathbb{C}$, $\mathrm{Im}(c) \neq 0$, is a root of $f(x)$, then there exists a real quadratic factor of $f(x)$ with c as a root.

Real Factors of Real Polynomials (RFRP)
Let $f(x) \in \mathbb{R}[x]$. Then $f(x)$ can be written as a product of real linear and real quadratic factors.