Solve $[15][x]+[7]=[12]$ in $\mathbb{Z}_{10}$.

Solution: This is equivalent to solving

$$15x+7 \equiv 12 \mod 10.$$

Isolating for x gives

$$15x \equiv 5 \mod 10.$$

Since $15 \equiv 5 \mod 10$, Properties of Congruences states that

$$5x \equiv 5 \mod 10.$$

This clearly has the solution $x=1$. Hence, by Linear Congruence Theorem 1, we have that

$$x \equiv 1 \mod \tfrac{10}{\gcd(5,10)}$$

gives the complete set of solutions. Thus, $x \equiv 1 \mod 2$ or $x \equiv 1,3,5,7,9 \mod 10$. Since the original question is framed in terms of congruence classes, our answer should be as well and hence

$$[x] \in \{[1],[3],[5],[7],[9]\}.$$

For extra practice, see if you can phrase this argument using Linear Congruence Theorem 2.