## Provided Reference for the Fall 2016 MATH 135 Midterm

(You do not need to submit this page with your exam.)

## A Note About Natural Numbers

Recall that for Math 135, we are using the notation

$$
\mathbb{N}=\{1,2,3,4, \ldots\}
$$

to denote the set of positive integers. This may be different from your CS course, where the set of natural numbers is often said to include zero as well.

## List of Propositions

You may use any of the results below without proof. When you do, make sure to clearly state the name (e.g. Transitivity of Divisibility) or the acronym (e.g. TD) associated with the result that you are using.

Transitivity of Divisibility (TD)
Let $a, b, c \in \mathbb{Z}$. If $a \mid b$ and $b \mid c$, then $a \mid c$.
Divisibility of Integer Combinations (DIC)
Let $a, b, c \in \mathbb{Z}$. If $a \mid b$ and $a \mid c$, then for all $x, y \in \mathbb{Z}, a \mid(b x+c y)$.
Bounds by Divisibility (BBD)
Let $a, b \in \mathbb{Z}$. If $a \mid b$ and $b \neq 0$, then $|a| \leq|b|$.

Division Algorithm (DA)
If $a \in \mathbb{Z}$ and $b \in \mathbb{N}$, then there exist unique integers $q$ and $r$ such that $a=q b+r$ where $0 \leq r<b$.

