L8P7
L9P1

List all elements of the set: If m is a positive divisor of n then m=1 or m=n.

$$S = \underbrace{\{n \in \mathbb{Z} : n > 1 \land ((m \in \mathbb{Z} \land m > 0 \land m \mid n) \Rightarrow (m = 1 \lor m = n))\}}_{\text{Set of all Primes!}}$$

$$\cap \underbrace{\{n \in \mathbb{Z} : n \mid 42\}}_{\{\cancel{\pm 1}, \pm 2, \pm 3, \cancel{\pm 6}, \pm 7, \cancel{\pm 14}, \cancel{\pm 21}, \cancel{\pm 42}\}}$$

Thus, $S = \{2, 3, 7\}$.

Rewrite the following using as few English words as possible.

1. No multiple of 15 plus any multiple of 6 equals 100.

2. Whenever three divides both the sum and difference of two integers, it also divides each of these integers.

1. $\forall m, n \in \mathbb{Z}, (15m + 6n \neq 100)$.

2. $\forall m, n \in \mathbb{Z} ((3 \mid (m+n) \wedge 3 \mid (m-n)) \Rightarrow 3 \mid m \wedge 3 \mid n)$.

Write the following statements in (mostly) plain English.

1. $\forall m \in \mathbb{Z}, ((\exists k \in \mathbb{Z}, m = 2k) \Rightarrow (\exists \ell \in \mathbb{Z}, 7m^2 + 4 = 2\ell))$

2. $n \in \mathbb{Z} \Rightarrow (\exists m \in \mathbb{Z}, m > n)$

1. If m is an even integer, then $7m^2+4$ is even.

2. For every integer, ~~there~~ there exists a greater integer.

There is no greatest integer.

Contrapositive.

Moral: Direct proofs are not always easy to find.

Eg: $7 \nmid n \Rightarrow 14 \nmid n \equiv 14 \mid n \Rightarrow 7 \mid n$.

Contrapositive Def'n:

The contrapositive of $H \Rightarrow C$ is $\neg C \Rightarrow \neg H$

Note: $H \Rightarrow C \equiv \neg C \Rightarrow \neg H$.

$$
\begin{aligned}
H \Rightarrow C \equiv \neg H \vee C &\equiv C \vee \neg H \\
&\equiv \neg(\neg C) \vee \neg H \\
&\equiv \neg C \Rightarrow \neg H .
\end{aligned}
$$

H	C	$H \Rightarrow C$	$\neg C$	$\neg H$	$\neg C \Rightarrow \neg H$
T	T	T	F	F	T
T	F	F	T	F	F
F	T	T	F	T	T
F	F	T	T	T	T

Ex: Let $x \in \mathbb{R}$. Prove $x^3 - 5x^2 + 3x \neq 15 \Rightarrow x \neq 5$.

Pf: We prove the contrapositive. Let $x = 5$. Then

$$
\begin{aligned}
x^3 - 5x^2 + 3x &= (5)^3 - 5(5)^2 + 3(5) \\
&= \cancel{5^3} - \cancel{5^3} + 15 \\
&= 15.
\end{aligned}
$$

Ex: Suppose $a, b \in \mathbb{R}$ and $ab \in \overbrace{\mathbb{R} - \mathbb{Q}}^{\text{Irrationals}}$. Show either $a \in \mathbb{R} - \mathbb{Q}$ <u>or</u> $b \in \mathbb{R} - \mathbb{Q}$.

Pf: Proceed by contrapositive. Suppose a is rational and b is rational. Then, $\exists\, k, l, m, n \in \mathbb{Z}$ s.t. $a = \frac{k}{l}$ and $b = \frac{m}{n}$ with $l, n \neq 0$. Then

$$ab = \frac{km}{ln} \in \mathbb{Q}.$$