Q1. I enjoy trying to discover and write MATH 135 proofs.

A) Strongly disagree

B) Disagree

C) Neither agree nor disagree

D) Agree

E) Strongly agree

Q2.When I have difficulties with MATH 135 proofs, I know I can handle them.

A) Strongly disagree

B) Disagree

C) Neither agree nor disagree

D) Agree

E) Strongly agree

Q3. Consider the following statement.

$$\{2k : k \in \mathbb{N}\} \supseteq \{n \in \mathbb{Z} : 8 \mid (n+4)\}$$

A well written and correct direct proof of this statement could begin with

A) We will show that the statement is true in both directions.

B) Assume that $8 \mid (n+4)$ where n is an integer. (CORRECT)

C) Let $m \in \{n \in \mathbb{Z} : 8 \mid (n+4)\}$.

D) Let $m \in \{2k : k \in \mathbb{N}\}$.

E) Assume that $8 \mid (2k+4)$.

Notes:

1. A single counter example proves that $(\forall x \in S, P(x))$ is false.

   Claim: Every positive even integer is composite.

   This claim is false since 2 is even but 2 is prime.

2. A single example does not prove that $(\forall x \in S, P(x))$ is true.

   Claim: Every even integer at least 4 is composite.

   This is true but we cannot prove it by saying "6 is an even integer and is composite." We must show this is true for an arbitrary even integer x. (Idea: $2 \mid x$ so there exists a $k \in \mathbb{N}$ such that $2k = x$ and $k \neq 1$.)

3. A single example does show that $(\exists x \in S, P(x))$ is true.

   Claim: Some even integer is prime.

   This claim is true since 2 is even and 2 is prime.

4. What about showing that $(\exists x \in S, P(x))$ is false?

   Idea: $(\exists x \in S, P(x))$ is false $\equiv \forall x \in S, \neg P(x)$ is true. This idea is central for proof by contradiction which we will see later.

Negating Quantifiers.

[?]: Negate the following.

[1]: Everybody in this room was born before 2010.

[Negation]: Somebody in this room was NOT born before 2010.

[2]: Someone in this room was born before 1990. (1987).

[Negate]: Everyone in this room was born after 1990.

[3]. $\forall x \in \mathbb{R}, |x| < 5$

[Negate]: $\exists x \in \mathbb{R}, |x| \geq 5 \equiv \neg(\forall x \in \mathbb{R}, |x| < 5)$

[4]. $\exists x \in \mathbb{R}, |x| \leq 5$

$\forall x \in \mathbb{R}, |x| > 5$.

[NB] A proof that a statement is false is called a disproof.

Q: Prove or disprove: Let $a,b,c \in \mathbb{Z}$. If $a|bc$ then $a|b \vee a|c$.

Soln: This is false! A counter example is given by $a=6$, $b=2$, $c=3$. Then $a|bc$. BUT $6\nmid 2$ and $6\nmid 3$.

Fix: Include that a must be prime. Proof is an exercise.

Which of the following are true?

1. $\forall x \in \mathbb{R}, \forall y \in \mathbb{R}, x^3 - y^3 = 1$ FALSE (Choose $x=0$, $y=0$)
2. $\exists x \in \mathbb{R}, \exists y \in \mathbb{R}, x^3 - y^3 = 1$ TRUE ($x=1$, $y=0$)
3. $\forall x \in \mathbb{R}, \exists y \in \mathbb{R}, x^3 - y^3 = 1$
4. $\exists x \in \mathbb{R}, \forall y \in \mathbb{R}, x^3 - y^3 = 1$

[3] TRUE: Pf: Let $x \in \mathbb{R}$ be arbitrary. Then choose $y = \sqrt[3]{x^3-1}$. Then

$$x^3 - y^3 = x^3 - \left(\sqrt[3]{x^3-1}\right)^3 = x^3 - (x^3-1) = 1.$$

∎.

[4] FALSE. Idea: Negate and show the negation is true.

$$\neg\left(\exists x \in \mathbb{R}, \forall y \in \mathbb{R}, x^3 - y^3 = 1\right)$$

$$\forall x \in \mathbb{R}, \exists y \in \mathbb{R}\ x^3 - y^3 \neq 1.$$

Let $x \in \mathbb{R}$ be arbitrary. Take $y = x$. Then

$$x^3 - y^3 = x^3 - x^3 = 0 \neq 1.$$

∎.

L8P6

Notation Cheat Sheet

1. $+$ Addition
2. $-$ Subtraction
3. $\times, \cdot$ Multiplication
4. $\div, /$ Division
5. $\mathbb{N}$ Natural Numbers
6. $\mathbb{Z}$ Integers (Zählen)
7. $\mathbb{Q}$ Rational Numbers (Quoziente)
8. $\mathbb{R}$ Real Numbers
9. $\neg$ Not, Negation
10. $\vee$ Or
11. $\wedge$ And
12. $|$ Divides
13. $\Rightarrow$ Implies (If... Then)
14. $\Leftrightarrow$, (iff) If and Only If
15. $\in$ In
16. $\notin$ Not In
17. $\{\}$, $\emptyset$ Empty Set
18. $\cap$ Intersection (Of Sets)
19. $\cup$ Union (Of Sets)
20. $\subset$ Subset
21. $\subseteq$ Subset Or Equal
22. $\subsetneq$ Proper/Strict Subset (Subset Not Equal)
23. $\supset$ Contains
24. $\supseteq$ Contains Or Equal
25. $\supsetneq$ Properly/Strictly Contains (Contains Not Equal)
26. $\forall$ For All
27. $\exists$ There Exists

List all elements of the set:

$$\{n \in \mathbb{Z} : n > 1 \wedge ((m \in \mathbb{Z} \wedge m > 0 \wedge m \mid n) \Rightarrow (m = 1 \vee m = n))\} \cap \{n \in \mathbb{Z} : n \mid 42\}$$