L6P0

Announcements:

A2 Print & Read! Due Wed. 8:25 AM

Check LEARN once a day for announcements.

Sets

$\{\}$ is different from $\{\emptyset\}$.

$\mathbb{Q} = \{ a/b \in \mathbb{R} : a, b \in \mathbb{Z}$ and $b \neq 0 \}$.

In the above example, $\mathbb{R}$ is called the "universe [of discourse]".

Ex: In set notation, write the set of positive integers less than 1000 and which are multiples of 7

Sol'n: $\{ n \in \mathbb{N} : n < 1000 \wedge 7 \mid n \}$.

$\{ 7k : k \in \mathbb{N}$ and $k \leq 142 \}$

↑ such that, s.t. t. |

Describe the following sets using set-builder notation:

1. Set of even numbers between 5 and 14 (inclusive).

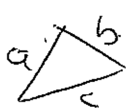
$\{6, 8, 10, 12, 14\}$ or $\{n \in \mathbb{N} : 5 \leq n \leq 14 \wedge 2 | n\}$

2. All odd perfect squares.

$\{(2k+1)^2 : k \in \mathbb{Z}\}$ $\boxed{\text{or } \mathbb{N}}$

3. Sets of three integers which are the side lengths of a (non-trivial) triangle.

$\frac{1 \quad 1}{2}$



$\{(a, b, c) : a, b, c \in \mathbb{N} \quad a < b + c \wedge b < a + c \wedge c < a + b\}$

4. All points on a circle of radius 8 centred at the origin.

$\{(x, y) : x, y \in \mathbb{R} \wedge x^2 + y^2 = 8^2\}$.

Set Operations:

Let S, T be sets. Define:

$S \cup T = \{x : x \in S \vee x \in T\}$ (union)

$S \cap T = \{x : x \in S \wedge x \in T\}$ (intersection)

$\bar{S}$ or S^c (with respect to universe $\mathcal{U}$)

$= \{x \in \mathcal{U} : x \notin S\} = \mathcal{U} - S$ (complement)

$S - T = \{x : x \in S \wedge x \notin T\}$ (set difference)

$S \times T = \{(x, y) : x \in S \wedge y \in T\}$. (Cartesian Product).

Ex. $(1,2) \in \mathbb{Z} \times \mathbb{Z}$, $(2,1) \in \mathbb{Z} \times \mathbb{Z}$

<u>BUT</u> $(1,2) \neq (2,1)$.

NB $\mathbb{Z} \times \mathbb{Z}$ and $\{(n,n) : n \in \mathbb{Z}\}$ are DIFFERENT <u>sets</u>!

Ex: $\mathbb{Z} = \overset{\text{EVEN}}{\{m \in \mathbb{Z} : 2 \mid m\}} \cup \overset{\text{ODD}}{\{2k+1 : k \in \mathbb{Z}\}}$.

& $\emptyset = \{m \in \mathbb{Z} : 2 \mid m\} \cap \{2k+1 : k \in \mathbb{Z}\}$

Def'n: $S \subseteq T$: S is a subset of T.

ie Every element of S is in T

$S \subsetneq T$: Proper / Strict subset

$S \supseteq T$: S contains T

$T \subseteq S$ ie Every element of T is in S.

$S \supsetneq T$: Proper / Strict containment.

S contains T AND $S \neq T$.

Def'n: $S = T$ means $S \subseteq T$ AND $T \subseteq S$.

Ex: $\{1,2\} = \{2,1\}$.

Ex: Prove $\{n \in \mathbb{N} : 4 \mid n+1\} \subseteq \{2k+1 : k \in \mathbb{Z}\}$

Pf: Let $m \in \{n \in \mathbb{N} : 4 \mid n+1\}$. Then $4 \mid m+1$. Thus, $\exists\, \ell \in \mathbb{Z}$ s.t. $4\ell = m+1$. Now, $m = 2(2\ell) - 1$

$$= 2(2\ell) - 2 + 2 - 1$$
$$= 2(2\ell - 1) + 1$$

Thus, $m \in \{2k+1 : k \in \mathbb{Z}\}$

Ex: Show $S=T$ iff $S \cap T = S \cup T$.

Pf: Suppose $S=T$. Then I claim $S \cap T = S$.
Now, $S \cap T \subseteq S$ since if $x \in S \cap T$, then by def'n $x \in S$. ~~Now~~ Similarly, if $x \in S$, then $x \in T$ (Since $S=T$) and thus $x \in S \cap T$.

Claim 2: $S \cup T = S$.

"$\supseteq$" is clear.

"$\subseteq$" Let $x \in S \cup T$ then either $x \in S$ and we are done OR $x \in T$ and since $S=T$, $x \in S$.

Thus, $S \cap T = S = S \cup T$.

For the converse, suppose $S \cap T = S \cup T$. ~~Let~~

Claim 3: $S \subseteq T$

Pf: Let $x \in S$. Then $x \in S \cup T = S \cap T$

So $x \in T$.

Claim 4: $T \subseteq S$.

Pf: Let $x \in T$. Then $x \in S \cup T = S \cap T$

So $x \in S$.

Claims 3 & 4 $\Rightarrow$ $S=T$

$\blacksquare$.