Office hours change: G227

M 10:00-11:00 AND 4:30 - 5:30.

R 1:00 - 2:00

F 1:00-2:00

A1 due 8:25 AM <u>WED</u>.

Divisibility of Integer Combinations (DIC)

If $a \mid b$ and $a \mid c$ then for all integers x, y we have $a \mid (bx+cy)$

Pf: Since $a|b$, $\exists k \in \mathbb{Z}$ s.t. $ak=b$.

Since $a|c$, $\exists l \in \mathbb{Z}$ s.t. $al=c$.

Then, $bx+cy = akx + aly$

$= a(kx+ly)$

Since $kx+ly \in \mathbb{Z}$, by def'n $a|(bx+cy)$ ∎.

Ex: Prove that if $m \in \mathbb{Z}$ and $14|m$ then $7|135m+693$

Pf: Suppose $m \in \mathbb{Z}$ and $14|m$. Since $7|14$ $(7 \cdot 2 = 14)$ by transitivity, $7|m$. As $7|693$ $(7 \cdot 99 = 693)$ we have, by DIC

$$7 \mid \underset{b}{m}\underset{x}{(135)} + \underset{c}{693}\underset{y}{(1)}$$

$$\Rightarrow 7 | 135m + 693$$

∎.

Converse

Def'n: Let A, B be statements. The converse of $A \Rightarrow B$ is $B \Rightarrow A$

Ex: If $p, p+1$ are prime, then $p=2$

Converse: If $p=2$ then $p, p+1$ are Prime.

(BBD) $a | b \wedge b \neq 0 \Rightarrow |a| \leq |b|$

Converse: $|a| \leq |b| \Rightarrow a | b \wedge b \neq 0$

NB: the converse is false!

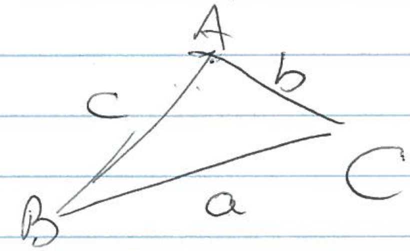
If and only if (iff)

Def'n: $A \Leftrightarrow B$, A iff B, A if and only if B

A	B	A <=> B
T	T	T
T	F	F
F	T	F
F	F	T

Ex: Show
$A \Leftrightarrow B$
$\equiv A \Rightarrow B \wedge B \Rightarrow A.$

Ex: In $\triangle ABC$, $b = c \cdot \cos A$ iff $\angle C = \frac{\pi}{2}$



Pf: Suppose $b = c \cdot \cos A$. By the cosine law,

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$a^2 = b^2 + c^2 - 2b \cdot b$$

$$a^2 = c^2 - b^2$$

$$a^2 + b^2 = c^2.$$

Using the cosine law again.

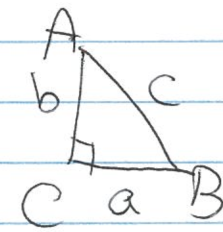
$$c^2 = a^2 + b^2 - 2ab\cos(\angle C)$$

$$\cancel{c^2} = \cancel{c^2} - 2ab\cos(\angle C)$$

$$0 = -2ab\cos(\angle C)$$

Thus $\cos(\angle C) = 0$. Since $0 < \text{angle } C < \pi$, we have $\angle C = \pi/2$.

Suppose now that $\angle C = \frac{\pi}{2}$.



Then, $\cos(A) = \frac{b}{c}$. Hence $c \cdot \cos A = b$ ∎.

Prove the following. Suppose $x, y \geq 0$. Show that $x = y$ if and only if $\frac{x+y}{2} = \sqrt{xy}$.

Suppose $\frac{x+y}{2} = \sqrt{xy}$

$x+y = 2\sqrt{xy}$

$x^2+2xy+y^2 = 4xy$

$x^2-2xy+y^2 = 0$

$(x-y)^2 = 0$.

Thus, $x-y=0 \Rightarrow x=y$.

Suppose $x=y$.

LHS $= \frac{x+y}{2}$

$= \frac{y+y}{2}$

$= \frac{2y}{2}$

$= y$.

RHS $= \sqrt{xy}$

$= \sqrt{y^2}$

$= y$ ($\because y \geq 0$)

LHS = RHS. $\boxtimes$

Set.

Def'n: A **set** is a collection of **elements**.

Eg: $\mathbb{Z}$, $\mathbb{N}$, $\mathbb{R}$, $\mathbb{Q}$ (set of rational numbers)

$\{5, A\}$, $S = \{1, 2, \diamond, \}$.

$x \in S$ x in S $x \notin S$ x not in S.

$\{\}$, $\emptyset$ empty set.

nota bene

NB: $\{\emptyset\}$ is NOT the same as the empty set.

This is a set that contains the empty set