Thurs. Dec. 11 4-6
Fri Dec. 12 ? 11-12
Sat. Dec. 13 2:30-4:30

Live Stream Friday 7:00.
Dec. 12.

Q1. I enjoy trying to discover and write MATH 135 proofs.

CODE
BC

A) Strongly disagree

B) Disagree

C) Neither agree nor disagree

D) Agree

E) Strongly agree

Q2. When I have difficulties with MATH 135 proofs, I know I can handle them.

A) Strongly disagree

B) Disagree

C) Neither agree nor disagree

D) Agree

E) Strongly agree

1. Let $a_0 = 1$ and $a_1 = 3$ and for $n \geq 2$, let $a_n = 3a_{n-1} - 2a_{n-2} - 1$. Show that $a_n = 2^n + n$ for all $n \geq 0$.

We prove the statement by strong induction on n

Base Case: $n = 0$

$a_0 = 1$ $\quad 2^n + n = 2^0 + 0 = 1$. ✓

$a_1 = 3$ $\quad 2^n + n = 2^1 + 1 = 3$ ✓

IH: Assume that $a_{\hat{k}} = 2^{\hat{k}} + \hat{k}$ for some $k \in \mathbb{Z}$, ~~$k \geq 0$~~. $k \geq 1$ for all $\hat{k} \in \{0, 1, \ldots k$

Istep: Want to prove that $a_{k+1} = 2^{k+1} + k + 1$ $\quad k \geq 1$

$$a_{k+1} = 3a_k - 2a_{k-1} - 1 \quad (\text{when } k+1 \geq 2)$$

$$\overset{IH}{=} 3(2^k + k) - 2(2^{k-1} + k - 1) - 1$$

$$= 3 \cdot 2^k + 3k - 2^k - 2k + 2 - 1$$

$$= 2 \cdot 2^k + k + 1 \quad \therefore \text{ by PosI the}$$

$$= 2^{k+1} + k + 1. \quad \text{statement is true } \forall n \in \mathbb{Z}, n \geq 0!$$

2.a. Let $a=-215$ and $b=17$. Find the integers q & r with $0 \le r < b$ s.t. $a=qb+r$

$$\begin{array}{r|l} & 12 \\ \hline 17 & 215 \\ & \underline{17} \\ & 45 \\ & \underline{34} \\ & 11 \end{array}$$

$215 = 12 \cdot 17 + 11$

$-215 = 17(-12) - 11$

$-215 = 17(-12) - 11 - 6 + 6$

$-215 = 17(-13) + 6$

b. List all pairs of integers (x,y) with $|x| \leq 50$ s.t. $245x + 189y = 84$.

x	y	r	q
1	0	245	
0	1	189	
1	-1	56	1
-3	4	21	3
7	-9	14	2
-10	13	7	1
x	x	0	2

$\therefore 245(-10) + 189(13) = 7.$

$245(-120) + 189(156) = 84$

GET $x = -120 + \frac{189}{7}n$ $\forall n \in \mathbb{Z}$.

$y = 156 - \frac{245}{7}n$

$x = -120 + 27n$

$y = 156 - 35n$ $\forall n \in \mathbb{Z}$

x	y
-120	156
-93	121
-66	86
-39	51
-12	16
15	-19
42	-54

$\therefore (x,y) \in \{(-39,51), (-12,16), (15,-19), (42,-54)\}$

$-50 \leq -120 + 27n \leq 50$

$70 \leq 27n \leq 170$