L39P1

Q1. I enjoy trying to discover and write MATH 135 proofs.

A) Strongly disagree

B) Disagree

C) Neither agree nor disagree

D) Agree

E) Strongly agree

Q2. When I have difficulties with MATH 135 proofs, I know I can handle them.

A) Strongly disagree

B) Disagree

C) Neither agree nor disagree

D) Agree

E) Strongly agree

Q3. What is the value of $\left|\left(\overline{-\sqrt{3}+i}\right)^5\right|$ ?

A) ~~$16i$~~

B) 27

C) 32

D) ~~$-45$~~

E) 64

$= |(-\sqrt{3}-i)^5|$

polar coord.

$= \left|\left(2\left(-\frac{\sqrt{3}}{2}-\frac{i}{2}\right)\right)^5\right|$

DMT $= |2^5 \operatorname{cis}(\frac{7\pi}{6}\cdot 5)|$

$= 32$

$|(-\sqrt{3}-i)|^5$

$= \sqrt{(-\sqrt{3})^2+(-1)^2}^{\,5}$

$= \sqrt{4}^{\,5}$

$= 32.$

Last Time: Solve $z^6 = -64$

- $z = re^{i\theta}$ $(r = |z|)$
- $64 = |z|^6 = r^6 |e^{i6\theta}| = r^6 \Rightarrow r = 2$
- $r^6 e^{i\cdot 6\theta} = -64 \Rightarrow e^{i6\theta} = -1$

$\cos(6\theta) + i\sin(6\theta) = -1 = \cos\pi + i\sin\pi$.

Equating real parts gives

$\cos(6\theta) = \cos(\pi) \Rightarrow 6\theta = \pi + 2\pi k$ (for $k \in \mathbb{Z}$)

Solving for $\theta$ gives: $\theta = \frac{\pi + 2\pi k}{6} = \frac{\pi}{6} + \frac{\pi}{3}k$.

When do two $\theta$ values coincide with the same complex point? A: When they differ by multiples of $2\pi$.

Claim: $\theta_1 = \frac{\pi}{6} + \frac{\pi}{3}k_1$ & $\theta_2 = \frac{\pi}{6} + \frac{\pi}{3}k_2$ are equal up to $2\pi$ rotations iff $k_1 \equiv k_2 \bmod 6$

Pf: $\theta_1 = \theta_2 + 2\pi m$ for some $m \in \mathbb{Z}$

$\Leftrightarrow \frac{\pi}{6} + \frac{\pi}{3}k_1 = \frac{\pi}{6} + \frac{\pi}{3}k_2 + 2\pi m$

$$\iff \frac{\pi}{3}k_1 = \frac{\pi}{3}k_2 + 2\pi m$$

$$\iff k_1 = k_2 + 6m$$

$$\iff k_1 \equiv k_2 \bmod 6.$$

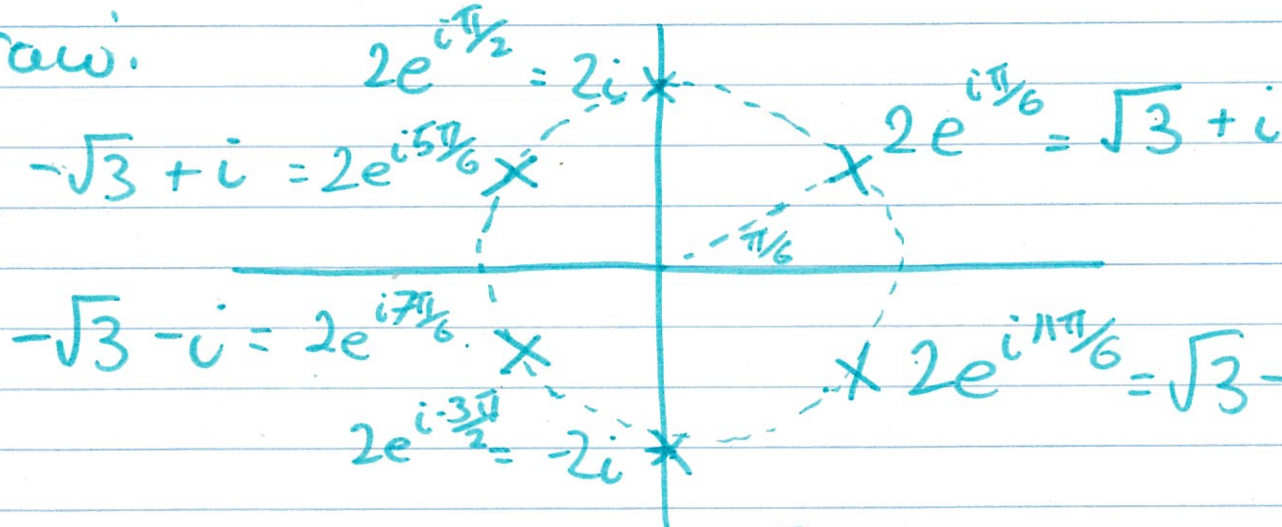
Hence $\theta = \frac{\pi}{6} + \frac{\pi}{3}k_1$ for $k_1 \in \{0,1,2,3,4,5\}$.

$\therefore \theta \in \{\frac{\pi}{6}, \frac{3\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{9\pi}{6}, \frac{11\pi}{6}\}$

$\theta \in \{\frac{\pi}{6} + \frac{\pi}{3}k_1 : k_1 \in \{0,1,2,3,4,5\}\}$.

$\therefore z = re^{i\theta} \in \{2e^{i(\frac{\pi}{6}+\frac{\pi}{3}k)} : k \in \{0,1,2,3,4,5\}\}$

Draw:




Complex $n^{th}$ Roots Theorem (CNRT)

Any non zero complex number has exactly $n \in \mathbb{N}$ distinct $n^{th}$ roots. The roots lie on a circle (of radius $|z|$).

centred at the origin and spaced out evenly by angles of $\frac{2\pi}{n}$.

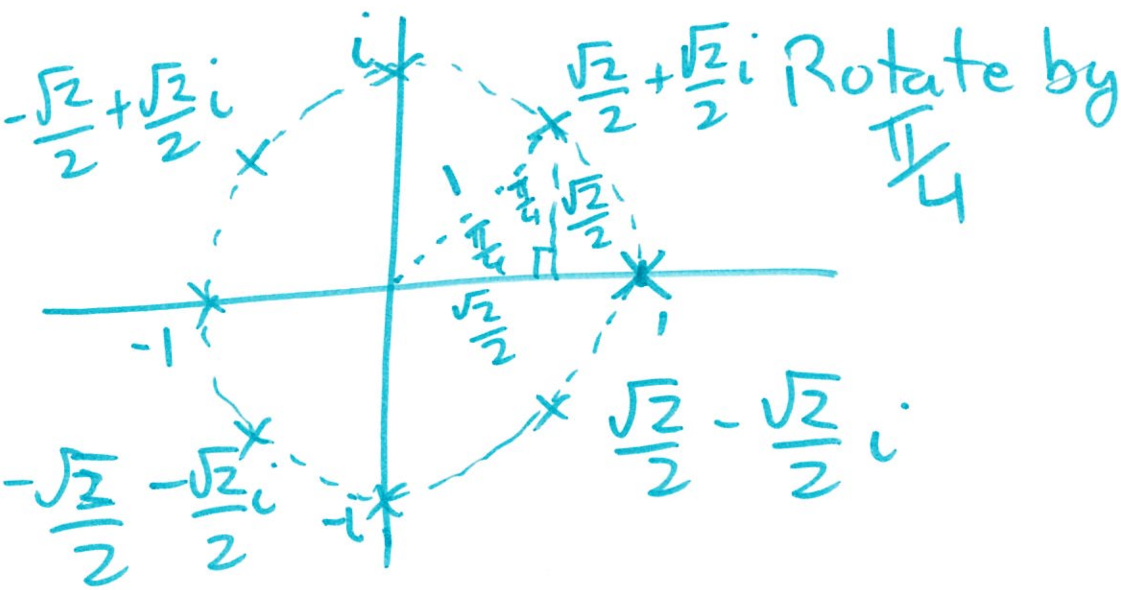
Def'n: An $n^{th}$ root of unity is a complex number $z$ s.t. $z^n = 1$. (Sometimes denoted by $\zeta_n$). (zeta)

Ex: $-1$ is a second root of unity (and fourth, and sixth, ...)

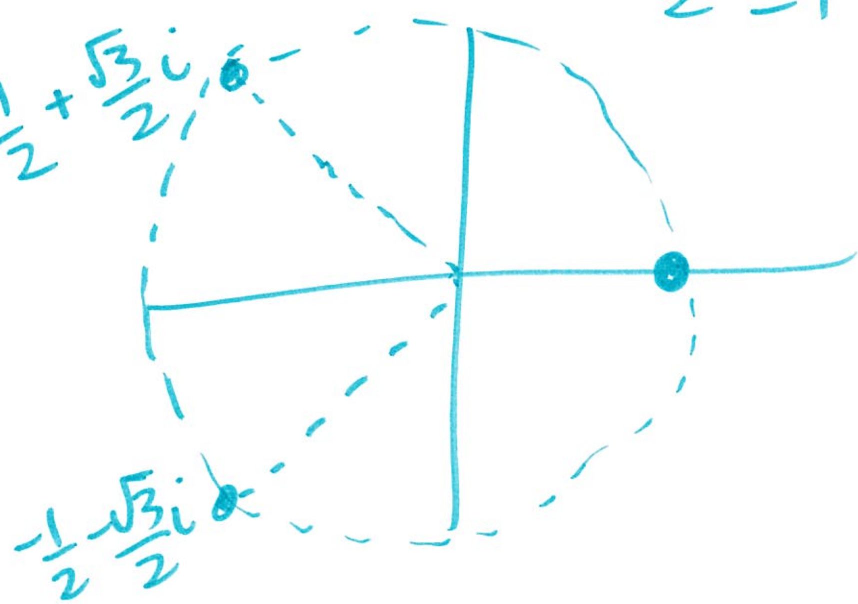
Find all eighth roots of unity in standard form. Draw.

Want to solve $z^8 = 1$

Know $z \in \{\pm 1, \pm i\}$ are solutions.




$z^3 = 1$



Solve

$$z^5 = -16\bar{z}$$

Sol'n: Tricky! Take moduli (by PM)

$$|z^5| = |z|^5 = |-16\bar{z}| = 16|\bar{z}| = 16|z|$$

$$|z|^5 = 16|z|$$

$$|z|^5 - 16|z| = 0$$

$$|z|(|z|^4 - 16) = 0$$

$$|z| = 0 \quad \underline{\text{OR}} \quad |z|^4 = 16.$$

$$\Leftrightarrow z = 0 \quad \underline{\text{OR}} \quad |z| = 2$$

Let's revisit $z^5 = -16\bar{z}$

Multiply by $z$: $z^6 = -16z\bar{z} = -16|z|^2 = -6$

$$\therefore z \in \{0, \pm 2i, \pm\sqrt{3} \pm i\}$$

7 solutions!