(DFPF)

Let $n>1$ be an integer and $d\in\mathbb{N}$. If $n=p_1^{\alpha_1}p_2^{\alpha_2}\cdots p_k^{\alpha_k}=\prod_{i=1}^{k}p_i^{\alpha_i}$ where $\alpha_i\in\mathbb{Z}$ are each ≥ 1, then d is a positive divisor of n iff a prime factorization of d is given by

$$d=p_1^{\delta_1}p_2^{\delta_2}\cdots p_k^{\delta_k}$$

where $\delta_i\in\mathbb{Z}$, $0\leq\delta_i\leq\alpha_i$ for $1\leq i\leq k$.

Ex: Divisors (positive) of $63=3^2\cdot 7$

$3^0\cdot 7^0,\ 3^0\cdot 7^1,\ 3^1\cdot 7^0,\ 3^1\cdot 7^1,\ 3^2\cdot 7^0,\ 3^2\cdot$

$1,\ 7,\ 3,\ 21,\ 9,\ 63.$

Pf is extra reading.

L22P2

How many multiples of 12 are positive divisors of 2940? What are they?

$$\begin{array}{r} 245 \\ 12\overline{)2940} \\ \underline{24} \\ 54 \\ \underline{48} \\ 60 \end{array}$$

$2940 = 12 \cdot 245$

$245 = 5 \cdot 49$

$= 5 \cdot 7^2$

Total number is $(1+1)(2+1) = 6$

Multiples are:

$12,\ 12\cdot 5,\ 12\cdot 7,\ 12\cdot 5\cdot 7,\ 12\cdot 7^2,\ 12\cdot 5\cdot 7^2$

Q: Prove $a^2 | b^2$ iff $a | b$.

Pf: Assume $a|b$. Then $\exists K \in \mathbb{Z}$ s.t. $aK = b$. Now, $a^2K^2 = b^2$ and hence $a^2 | b^2$ by def'n.

Now, assume $a^2 | b^2$. For convenience, $a, b > 0$. Write (Exer: prove true when $a \in \{0, \pm 1\}$)

$$a = \prod_{i=1}^{K} p_i^{\alpha_i} \qquad b = \prod_{i=1}^{K} p_i^{\beta_i}$$

with $0 \le \alpha_i, \beta_i$ integers. Since $a^2 | b^2$, $\prod_{i=1}^{K} p_i^{2\alpha_i} \,\Big|\, \prod_{i=1}^{K} p_i^{2\beta_i}$. DF?F implies $2\alpha_i \le 2\beta_i \Rightarrow \alpha_i \le \beta_i$ for $1 \le i \le K$. DFPF again implies

$$a = \prod_{i=1}^{K} p_i^{\alpha_i} \,\Big|\, \prod_{i=1}^{K} p_i^{\beta_i} = b.$$

$\blacksquare$.

GCDPF

Ex: $\gcd(2^5 \cdot 3^0 \cdot 5^4, 2^4 \cdot 3^1 \cdot 5^4)$
$= 2^{\min\{4,5\}} \cdot 3^{\min\{0,1\}} \cdot 5^{\min\{4,4\}}$
$= 2^4 \cdot 5^4 = 10000$

Statement: If

$$a = \prod_{i=1}^{\ell} p_i^{\alpha_i} \quad \text{and} \quad b = \prod_{i=1}^{\ell} p_i^{\beta_i}$$

where $0 \le \alpha_i, \beta_i$ are integers and p_i are <u>distinct</u> primes. Then

$$\gcd(a,b) = \prod_{i=1}^{\ell} p_i^{m_i}$$

where $m_i = \min\{\alpha_i, \beta_i\}$ for $1 \le i \le \ell$

Pf is extra reading.

Let $lcm(a,b)$ represent the least common multiple of a & b.

Ex: 1. Write a formal defn for $lcm(a,b)$

2. Show
$$lcm(a,b) = \prod_{i=1}^{\ell} p_i^{e_i}$$
where $e_i = \max\{\alpha_i, \beta_i\}$

3. Prove $gcd(a,b) \cdot lcm(a,b) = ab$

L22 P6
(EXTRA)

Linear Diophantine Equations.

Common DE: $x^2+y^2=z^2$ (NOT Linear)

Pythagoreen triples

$$\left.\begin{array}{l} ax=b \\ ax+by=c=\gcd(a,b) \end{array}\right\} \text{Linear.}$$

$$56x+249y=31$$

$$2x+4y=3$$