Def'n: For $x \in \mathbb{R}$, define the floor function $\lfloor x \rfloor$ to be the greatest integer less than or equal to x.

Ex: $\lfloor 2.5 \rfloor = 2 = \lfloor 2 \rfloor$

$\lfloor \pi \rfloor = 3$ $\qquad$ $\lfloor 0 \rfloor = 0$

$\lfloor -2.5 \rfloor = -3$

//

Find $\gcd(56, 35)$

[1] $\qquad 56(1) + 35(0) = 56$

[2] $\qquad 56(0) + 35(1) = 35$ $\qquad q_1 = \lfloor \frac{56}{35} \rfloor = 1$

[3] = [1] − q_1[2] $\qquad 56(1) + 35(-1) = 21$ $\qquad q_2 = \lfloor \frac{35}{21} \rfloor = 1$

[4] = [2] − q_2[3] $\qquad 56(-1) + 35(2) = 14$ $\qquad q_3 = \lfloor \frac{21}{14} \rfloor = 1$

[5] = [3] − q_3[4] $\qquad 56(2) + 35(-3) = 7$ $\qquad q_4 = \lfloor \frac{14}{7} \rfloor = 2$

[6] = [4] − q_4[5] $\qquad 56(-5) + 35(8) = 0.$

$\therefore \gcd(56, 35) = 7 = 56(2) + 35(-3).$

Ex: Find $x, y \in \mathbb{Z}$ s.t.

$$506x + 391y = \gcd(506, 391)$$

	x	y	r	q
[1]	1	0	506	0
[2]	0	1	391	0
[3]=[1]-[2]	1	-1	115	$\lfloor \frac{506}{391} \rfloor = 1$
[4]=[2]-3[3]	-3	4	46	$\lfloor \frac{391}{115} \rfloor = 3$
[5]=[3]-2[4]	7	-9	23	$\lfloor \frac{115}{46} \rfloor = 2$
[6]=[4]-2[5]	-17	22	0	$\lfloor \frac{46}{23} \rfloor = 2$

$\therefore\ 506(7) + 391(-9) = 23 = \gcd(506, 391)$

This is called the Extended Euclidean Algorithm (EEA).

Use the Extended Euclidean Algorithm to find integers x and y such that $408x + 170y = \gcd(408, 170)$.

x	y	r	q
1	0	408	0
0	1	170	0.
1	-2	68	2
-2	5	34	$\lfloor \frac{170}{68} \rfloor = 2$
5	-12	0	$\frac{68}{34} = 2$

$\therefore 408(-2) + 170(5) = 34 = \gcd(408, 170)$

Quick Notes:

- Bézout's Lemma is EEA in textbook.
- With gcd(a,b) what if
  - $b > a$? Swap a & b. Works since $\gcd(a,b) = \gcd(b,a)$.
  - What if $a < 0$ or $b < 0$?

    Sol'n: Make it positive. Works since

    $$\gcd(a,b) = \gcd(-a,b) = \gcd(a,-b) = \gcd(-a,-b).$$

L20 P5

Use the Extended Euclidean Algorithm to find integers x and y such that $399x - 2145y = \gcd(399, -2145)$.

Find $\tilde{x}, \tilde{y} \in \mathbb{Z}$ s.t. ~~$399\tilde{x} + 2$~~

$2145\tilde{x} + 399\tilde{y} = \gcd(2145, 399)$

$\tilde{x}$	$\tilde{y}$	r	q
1	0	2145	0
0	1	399	0
1	-5	150	5
-2	11	99	2
3	-16	51	1
-5	27	48	1
8	-43	3	1
-5-16(8)	27-(16)(-43)	0	16

$\therefore \quad 2145(8) + 399(-43) = 3 = \gcd(2145, 399)$

$\therefore \quad -2145(-8) + 399(-43) = 3 = \gcd(399, -2145)$