

# Trigonometric Substitution

Goal: Integrate certain radical (square root) functions.

Let's begin by motivating this using the area of a quarter circle.

**Example** Let  $r$  be a positive number. Evaluate  $4 \int_0^r \sqrt{r^2 - x^2} dx$

Table of substitutions. When integrating a function with a term in the left most column, do the substitutions in the other columns.

Expression	Substitution (interval where function is one to one)	Identity
$\sqrt{a^2 - x^2}$	$x = a \sin(\theta) \quad -\pi/2 \leq \theta \leq \pi/2$	$1 - \sin^2(\theta) = \cos^2(\theta)$
$\sqrt{a^2 + x^2}$	$x = a \tan(\theta) \quad -\pi/2 < \theta < \pi/2$	$1 + \tan^2(\theta) = \sec^2(\theta)$
$\sqrt{x^2 - a^2}$	$x = a \sec(\theta) \quad 0 \leq \theta < \pi/2 \text{ or } \pi \leq \theta < 3\pi/2$	$\sec^2(\theta) - 1 = \tan^2(\theta)$

Here are some examples

(i)  $\int \frac{dx}{x^2+4x+7}$

(ii)  $\int \frac{2x}{\sqrt{x^2+1}} dx$

(iii)  $\int \frac{dx}{x^2\sqrt{x^2+16}}$