Decay: We want a pdf which represents atom decay. Turns out

$$p(t) = C e^{-Kt} \qquad K \in \mathbb{R} \text{ given constant for } t \geq 0$$

is a good candidate $0 \leq t$

(a) Find the normalization constant C. $(C = K)$

(b) Compute the mean & variance. $\left(\mu = \frac{1}{K} \quad V = \frac{2}{K^2} - \frac{1}{K}\right)$

(a) $$1 = \int_0^\infty C e^{-Kt}\,dt = \lim_{b \to \infty} \int_0^b C e^{-Kt}\,dt = \lim_{b \to \infty} \frac{C e^{-Kt}}{-K}\Big|_0^b$$

$$= \lim_{b \to \infty} \frac{C e^{-Kb}}{K} - \frac{C e^{-K(0)}}{-K} = \frac{C}{K} \Rightarrow C = K.$$

Mean: $$\int_0^\infty t K e^{-Kt}\,dt = \frac{1}{K} \qquad \text{(Integration by parts).}$$

Integration by parts twice.

Variance: $$\int_0^\infty t^2 K e^{-Kt}\,dt - \int_0^\infty t K e^{-Kt}\,dt = \frac{2}{K^2} - \frac{1}{K}$$

$$\int e^{\sqrt{x}}\,dx \qquad u = \sqrt{x}, \quad du = \frac{1}{2\sqrt{x}}\,dx$$

$$= \int 2u e^{u}\,du \qquad v = u, \quad w = e^{u}, \quad dv = du, \quad dw = e^{u}\,du$$

$$= 2\left(ue^{u} - \int e^{u}\,du\right)$$

$$= 2ue^{u} - 2e^{u} + C$$

$$= 2\sqrt{x}\,e^{\sqrt{x}} - 2e^{\sqrt{x}} + C$$

$$\int_0^1 \frac{2x}{\sqrt{x^2+1}}\,dx \qquad u = x^2+1, \quad du = 2x\,dx, \qquad u(0) = 1, \quad u(1) = 2$$

$$= \int_1^2 \frac{du}{\sqrt{u}} = 2u^{\frac{1}{2}}\Big|_1^2 = 2^{\frac{3}{2}} - 2.$$

$$\int \frac{4x+2}{1-x^2}\,dx \qquad 4x+2 = A(1-x) + B(1+x)$$

plug in x=1 $B = 3$, $2B \stackrel{!}{=} 6$ $\qquad A = -1$ plug in x=−1 $\qquad 2A = -4+2 = -2$

$$\cancel{= -\ln|1-x| + 3\ln|1+x| + C}$$

$$\frac{4x+2}{1-x^2} = \frac{A}{1+x} + \frac{B}{1-x} = \frac{A(1-x) + B(1+x)}{1-x^2}$$

$$\int \frac{4x+2}{1-x^2}\,dx = \int\left(\frac{-1}{1+x} + \frac{3}{1-x}\right)dx = -\ln|1+x| + 3\left(-\ln|1-x|\right) + C$$

Find the mean and median of $\frac{1}{1-e^{-2}} e^{-x}$ (on $[0,2]$).

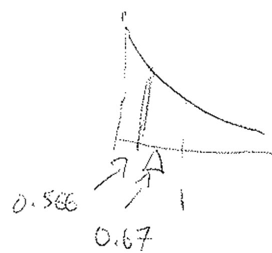
$$\bar{x} = \int_0^2 x\left(\frac{1}{1-e^{-2}} e^{-x}\right)dx = \frac{1}{1-e^{-2}}\int_0^2 xe^{-x}\,dx \qquad \begin{array}{ll} u = x & v = -e^{-x} \\ du = dx & dv = e^{-x}dx \end{array}$$

$$= \frac{1}{1-e^{-2}}\left(-xe^{-x}\Big|_0^2 - \int_0^2 -e^{-x}\,dx\right)$$

$$= \frac{1}{1-e^{-2}}\left(-2e^{-2} - 0 - \left(e^{-x}\right)\Big|_0^2\right)$$

$$= \frac{1}{1-e^{-2}}\left(-2e^{-2} - e^{-2} + e^{0}\right)$$

$$= \frac{1-3e^{-2}}{1-e^{-2}} \approx 0.67.$$



Median: Want x_{med} such that $F(x_{med}) = \frac{1}{2}$ (cdf). Recall our cdf is $F(x) = \frac{1-e^{-x}}{1-e^{-2}}$

So we want.

$$\frac{1-e^{-x_{med}}}{1-e^{-2}} = \frac{1}{2}$$

$$2 - 2e^{-x_{med}} = 1 - e^{-2}$$

$$\frac{1+e^{-2}}{2} = \frac{2e^{-x_{med}}}{2}$$

$$0.566 \approx \ln\left(\frac{2}{1+e^{-2}}\right) = -\ln\left(\frac{1+e^{-2}}{2}\right) = x_{med}.$$

Mode is at 0 since $p(x)$ is decreasing.

Evaluate $\int (\ln x)^2 \, dx$

(A) $x\ln x - x + C$
(B) $x^2 \ln^2 x - x^2 + C$
(C) $x\ln^2 x - \ln x + x + C$
(D) $x\ln^2 x - 2x\ln x + 2x + C$ ←
(E) other.

$$\int (\ln x)^2 dx$$
$$= x(\ln x)^2 - \int 2\ln x \, dx$$
$$= x\ln^2 x - 2\left(x\ln x - \int dx\right)$$
$$= x\ln^2 x - 2x\ln x + 2x + C.$$

$u = (\ln x)^2 \quad v = x$
$du = \frac{2\ln x}{x} dx \quad dv = dx$

$\hat{u} = \ln x \quad \hat{v} = x$
$d\hat{u} = \frac{dx}{x} \quad d\hat{v} = dx$

$$\int 4x^2\sqrt{x^3+2}\,dx$$

$u = x^3+2$

$du = 3x^2\,dx$

$$= \int 4\sqrt{u}\left(\frac{du}{3}\right)$$

$$= \left(\frac{4}{3}\right)\left(\frac{2u^{3/2}}{3}\right) + C$$

$$= \frac{8}{9}(x^3+2)^{3/2} + C$$

$1 + \tan^2\theta = \sec^2\theta$

$$\int_0^{11} \frac{dx}{\sqrt{225+x^2}}$$

$x = 15\tan\theta$ $\qquad \sqrt{225} = 15$

$0 = 15\tan\theta$ so $\theta = 0$

$11 = 15\tan\theta$ so $\theta = \arctan\left(\frac{11}{15}\right)$ $\quad dx = 15\sec^2\theta\,d\theta$

$$= \int_0^{\arctan(\frac{11}{15})} \frac{15\sec^2\theta\,d\theta}{\sqrt{225+225\tan^2\theta}}$$

$$= \int_0^{\arctan(\frac{11}{15})} \frac{15\sec^2\theta\,d\theta}{\sqrt{225}\sqrt{1+\tan^2\theta}} = \int_0^{\arctan(\frac{11}{15})} \frac{\sec^2\theta}{\sqrt{\sec^2\theta}}\,d\theta$$

$$= \int_0^{\arctan(\frac{11}{15})} \sec\theta\,d\theta$$

$$= \int_0^{\arctan(\frac{11}{15})} \sec\theta\,\frac{(\sec\theta+\tan\theta)}{(\sec\theta+\tan\theta)}\,d\theta$$

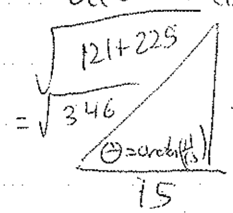
$$= \int_0^{\arctan(\frac{11}{15})} \frac{\sec^2\theta+\sec\theta\tan\theta}{\sec\theta+\tan\theta}\,d\theta$$

Let $u = \sec\theta + \tan\theta$

$du = (\sec\theta\tan\theta + \sec^2\theta)\,d\theta$

$u(0) = \sec(0) + \tan(0) = 1$

$u\left(\arctan\left(\frac{11}{15}\right)\right) = \boxed{\frac{\sqrt{346}}{15} + \frac{11}{15}}$



$$= \int_1^{\frac{\sqrt{346}+11}{15}} \frac{du}{u}$$

$$= \ln|u| + C$$

$$= \ln\left(\frac{\sqrt{346}+11}{15}\right) \cancel{- \ln(1)}$$

$$1-\int_0^t p(x)\,dx \qquad \int_t^\infty p(x)\,dx$$

$$= 1 - \int_0^{\hat{t}} ce^{-ct}\,dt \qquad \int_{\hat{t}}^\infty ce^{-ct}\,dt$$

$$= 1 + \left(e^{-ct}\right)\Big|_0^{\hat{t}}$$

$$= 1 + e^{-c\hat{t}} - (e^0)$$

$$p(t) = ce^{-ct} \quad [0,\infty)$$

$$= e^{-c\hat{t}}$$

$$S(3) = 0.55$$

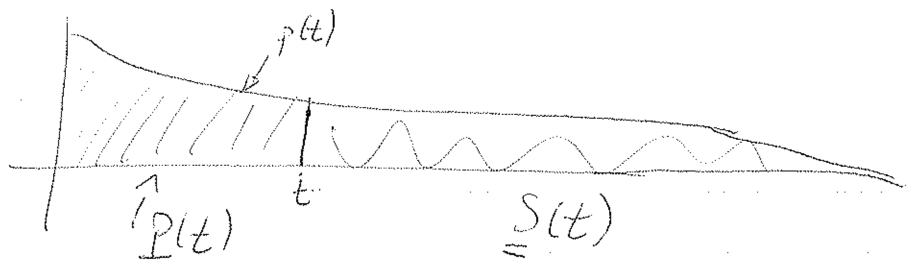
$$S(\hat{t}) = e^{-c\hat{t}}$$

$$e^{-c3} = 0.55$$

$$c = \frac{-\ln 0.55}{3}$$

$$S(z) = e^{-cz}$$

$$\bar{x} = \int_0^\infty x p(x)\,dx \qquad P(4).$$



$$\int \frac{4x+2}{1-x^2}\,dx.$$

$$\int 2t \log_3(t)\,dt.$$

$$= \int 2t\,\frac{\ln(t)}{\ln(3)}\,dt$$

$$= \frac{2}{\ln(3)} \int t\ln(t)\,dt.$$

$u = \ln t \qquad v = \frac{t^2}{2} dt$

$du = \frac{dt}{t} \qquad dv = t\,dt$

$$= \frac{2}{\ln 3}\left(\frac{t^2}{2}\ln t - \int \frac{t^2}{2}\,\frac{dt}{t}\right)$$

$$= \frac{2}{\ln 3}\left(\frac{t^2 \ln t}{2} - \frac{t^2}{4}\right) + C$$

$$\int_0^1 \left(\sum_{n=1}^{10} x^n\right) dx$$

$$\int (x+5)\,dx = \int x\,dx + \int 5\,dx$$

$$\int_0^1 (x^1 + x^2 + x^3 + x^4 + \cdots + x^{10})\,dx$$

linearity.

$$= \sum_{n=1}^{10} \int_0^1 x^n\,dx$$

$$\int_0^1 x^1 + \int_0^1 x^2 + \int_0^1 x^3 + \cdots \int_0^1 x^{10}\,dx$$

$$= \sum_{n=1}^{10} \frac{x^{n+1}}{n+1}\Big|_0^1$$

$$= \sum_{n=1}^{10} \frac{1}{n+1} = \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{11}.$$

$$\int_0^{2\pi} \sin x \cdot \sin(x+1)\,dx$$

Trig identity
$2\cos^2\alpha - 1 = \cos(2\alpha)$
$2(1-\sin^2\alpha) - 1 = \cos(2\alpha)$
$1 - 2\sin^2\alpha = \cos(2\alpha)$
$\frac{1-\cos(2\alpha)}{2} = \sin^2\alpha$

$$= \int_0^{2\pi} \sin x\,(\sin(x)\cos(1) + \sin(1)\cos(x))$$

$$= \int_0^{2\pi} \cos(1)\sin^2(x)\,dx + \sin(1)\int_0^{2\pi} \sin(x)\cos(x)\,dx$$

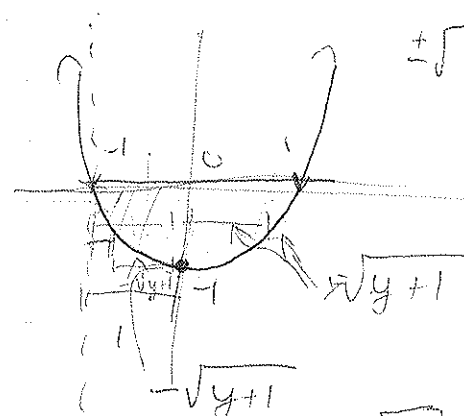
$$= \cos(1)\int_0^{2\pi} \sin^2(x)\,dx + \sin(1)\int_0^{2\pi} \frac{\sin(2x)}{2}\,dx$$

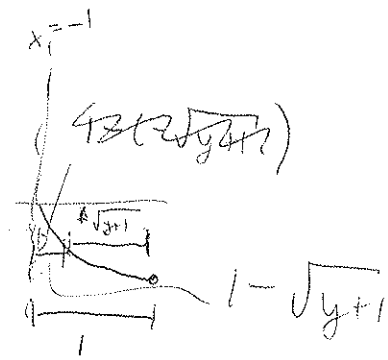
$$= \cos(1)\int_0^{2\pi} \left(\frac{1-\cos(2x)}{2}\right)dx + \sin(1)\int_0^{2\pi} \frac{\sin(2x)}{2}\,dx$$

$$= \cos(1)\left(\left(x - \frac{\sin(2x)}{4}\right)\Big|_0^{2\pi}\right) + \sin(1)\left(-\frac{\cos(2x)}{4}\Big|_0^{2\pi}\right)$$

$y = x^2 - 1$ about $x = -1$

$y+1 = x^2$
$\pm\sqrt{y+1} = x$





$1-\sqrt{y+1}$

$$\int_{-1}^{0}\Big(\underbrace{\pi\left(\sqrt{y+1}+1\right)^2}_{\text{Rightmost curve rotated.}} - \pi\left(1-\sqrt{y+1}\right)^2\Big)\,dy$$

$2\sqrt{y+1}$ $\quad$ $-2\sqrt{y+1}$

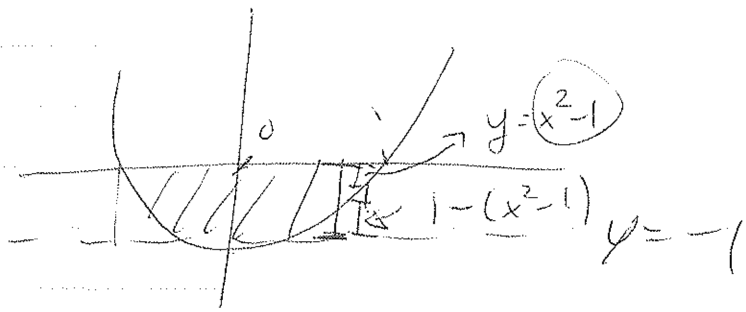
$$\pi(1 + (y+1)) - \pi(1 + (y+1))$$

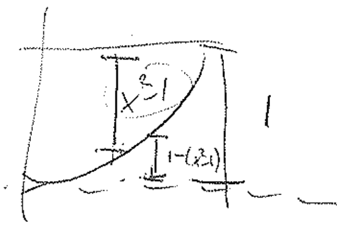
$+2y\pi\Big|_1^0$

$$\int_{-1}^{0} 4\pi\sqrt{y+1}$$

$$\frac{4\pi\sqrt{y+1}^{\,3}}{3}\Big|_0^1$$

$$\frac{8\pi}{3}$$





$$= 2\int_0^1 \pi(1)^2 - \pi\left(1-(x^2-1)\right)^2\,dx$$

NB Double check revolution.