Probability

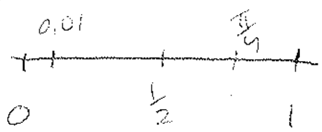
Discrete probability: Let X & Y be random variables.

For example X & Y can be used to represent dice rolls.
Let X represent rolling a D6 die (cube)
" Y " " " D20 die (icosahedron)

What is $Pr(X=4)$? $= \frac{1}{6}$
$Pr(Y=4)$? $= \frac{1}{20}$
$Pr(X=4 \text{ and } Y=4)$? $= \frac{1}{6} \cdot \frac{1}{20} = \frac{1}{120}$
$Pr(X=7)$? $= 0$.

Discrete probability: Probability of an event $= \frac{\#\text{ of successes}}{\text{total }\#}$.

Thought experiment:

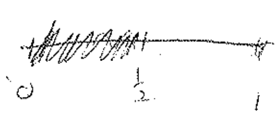


Q: What is the probability of randomly choosing $\frac{1}{2}$ if you pick a real number between 0 and 1? Ans: 0.

Paradox: Probability 0 events CAN occur (if your sample space is infinite)

Idea: Switch to thinking about lengths & mass.

Back to Dice ex: $Pr(0 \leq X \leq 3)$? $\frac{1}{2}$. $\left(\frac{3}{6}\right)$



Pr(I choose a number between 0 and 1 at random from 0 to $\frac{1}{2}$)? $= \frac{1}{2}$

L'Hôpital's Rule

Thm: Let $f(x)$ & $g(x)$ be differentiable functions. Assume that ~~both~~ either

$$\lim_{x\to a} f(x) = \lim_{x\to a} g(x) = 0 \text{ or } \pm\infty. \qquad a\in\mathbb{R}\cup\{\pm\infty\}$$

~~Then~~ and further that $\lim_{x\to a} \dfrac{f'(x)}{g'(x)}$ exists. Then

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}.$$

Ex: $\lim_{x\to 1} \dfrac{\ln x}{x-1}$ $\qquad \lim_{x\to 1} \ln x = 0$ & $\lim_{x\to 1} x-1 = 0$.

$$\overset{LH}{=} \lim_{x\to 1} \frac{\overbrace{\tfrac{1}{x}}^{\frac{d}{dx}\ln x}}{\underbrace{1}_{\frac{d}{dx}(x-1)}} = 1.$$

Ex: $$\lim_{x\to\infty} \frac{\ln x}{x^3} \overset{\frac{\infty}{\infty} LH}{=} \lim_{x\to\infty} \frac{\frac{1}{x}}{3x^2} = \lim_{x\to\infty} \frac{1}{x} \div 3x^2 = \lim_{x\to\infty} \frac{1}{3x^3} = 0.$$

Ex: $$\lim_{x\to\infty} \frac{x^2}{e^x} \overset{\frac{\infty}{\infty} LH}{=} \lim_{x\to\infty} \frac{2x}{e^x} \overset{\frac{\infty}{\infty} LH}{=} \lim_{x\to\infty} \frac{2}{e^x} = 0.$$

$$\lim_{x\to\infty} \frac{e^x}{x^2} \overset{\frac{\infty}{\infty} LH}{=} \lim_{x\to\infty} \frac{e^x}{2x} \overset{\frac{\infty}{\infty} LH}{=} \lim_{x\to\infty} \frac{e^x}{2} = \infty$$

Ex: $\lim_{x\to\infty} x\ln(1+x^{-1})$ this is $0\cdot\infty$ so write $x = \frac{1}{x^{-1}}$

$$= \lim_{x\to\infty} \frac{\ln(1+x^{-1})}{x^{-1}} \overset{\frac{\infty}{\infty}}{=} \lim_{x\to\infty} \frac{\frac{x}{x}\cdot\frac{1}{1+x^{-1}}(-x^{-2})}{(-x^{-2})} = \lim_{x\to\infty} \frac{x}{x+1} = 0.$$