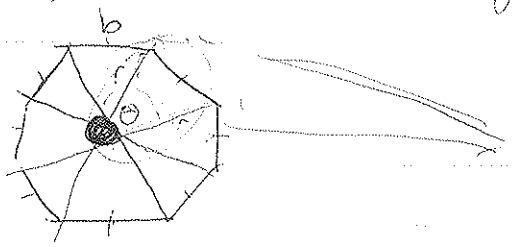
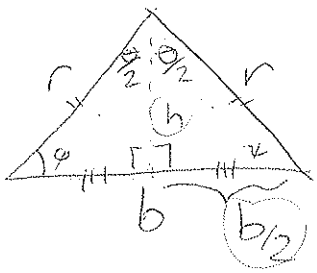
Jan 7/14

Chapter 1 Area of an "n"-gon of side length "b"

(Regular "n"-gon i.e. All side lengths are the same)

n=8-gon





There are n of these triangle

"n" triangles

$\theta = \frac{2\pi}{8} = \frac{\pi}{4}$ in general $\theta = \frac{2\pi}{n}$

Area of "n"-gon $= n\left(\frac{bh}{2}\right) = \frac{nb}{2} \cdot \frac{b}{2\tan(\theta/2)} = \frac{nb^2}{4\tan(\pi/n)}$

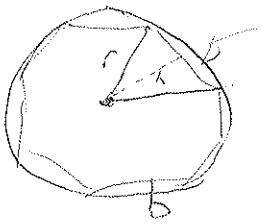
Goal: write h in terms of b and n.

$\tan(\theta/2) = \frac{b/2}{h}$ so $h = \frac{b}{2\tan(\theta/2)}$

Ex: Area of unit square is $\frac{4 \cdot 1^2}{4 \cdot \tan(\frac{\pi}{4})} = 1$

Area of a circle.

Let. $\pi = \dfrac{\text{Circumference of circle}}{\text{diameter of a circle}} = \dfrac{C}{d} = \dfrac{C}{2r}$



n-gon

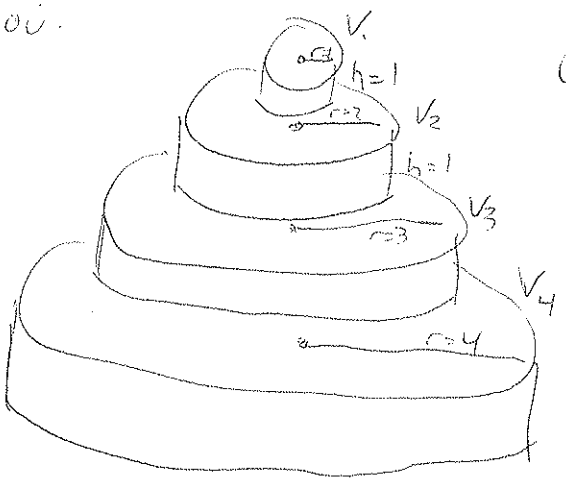
$A_{circle} = \lim_{n\to\infty} A_{n\text{-gon}} = \lim_{n\to\infty} \dfrac{nbh}{2}$

$= \left(\dfrac{(2\pi r)r}{2}\right) = \pi r^2.$

As $n \to \infty$ $\quad h \xrightarrow{n\to\infty} r.$

Perimeter of "n-gon $= nb \Rightarrow$ ~~Perimeter~~ circumference of circle $= 2\pi r.$

Tower of Hanoi.



Q: Compute the volume of tower.

$V_{cylinder} = \pi r^2 h$

$V_{Tower} = V_1 + V_2 + V_3 + V_4 = \pi(1)^2 h + \pi(2)^2 \cdot h + \pi(3)^2 \cdot h + \pi(4)^2 \cdot h$

$= \pi + 4\pi + 9\pi + 16\pi \qquad (h = 1 \text{ always}).$

$= 30\pi$

Q: Compute the volume of a 100-disc tower.

$V_{Tower} = V_1 + V_2 + \cdots + V_{100}$

$= \pi(1)^2 \cdot 1 + \pi(2)^2 \cdot 1 + \cdots + \pi(100)^2 \cdot 1.$

Sigma Notation

• A compact way to write summations

Let $\{a_i\}_{i=1}^{n}$ be a sequence of n real numbers (n is a positive integer)

Write $\sum_{i=1}^{n} a_i \overset{\text{def}}{=} a_1 + a_2 + \cdots + a_n$

(n ← end index; $i=1$ ← start index)

Ex: $\sum_{i=2}^{5} (i+1)$ (write in standard notation Learning goal 1.4).

$= ((2)+1) + ((3)+1) + ((4)+1) + ((5)+1)$

($i=2$ $\quad i=3$ $\quad i=4$ $\quad i=5$ $\quad i=6>5$ so stop!)

$= 3+4+5+6$

$= 18$

Ex: Write in sigma notation the sum of the first 10 natural numbers.

$$\sum_{i=1}^{10} i = 1+2+\cdots+10 = 55$$

Ex: Write in sigma notation

$$5+8+11+14+17+20 \overset{\text{Answer}}{=} \sum_{i=1}^{6} 3i+2 = \sum_{j=0}^{5} 3j+5$$

Ex: $\sum_{i=-2}^{-1} (i+1) = (-2+1)+(-1+1) = -1$

Ex: Let f be a function

$$\sum_{k=1}^{3} f(k) = f(1) + f(2) + f(3).$$

Ex: Write Infinite sums.

$$1 - 2 + 3 - 4 + 5 - 6 + \cdots = \sum_{j=1}^{\infty} (-1)^{j+1} j = \sum_{j \geq 1} (-1)^{j+1} j.$$

Special Sums: Let n be a positive integer

$$\sum_{j=1}^{n} j = 1 + 2 + \cdots + n = \frac{n(n+1)}{2}$$

$$\sum_{j=1}^{n} j^2 = 1^2 + 2^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{j=1}^{n} j^3 = 1^3 + 2^3 + \cdots + n^3 = \left(\frac{n(n+1)}{2}\right)^2.$$