In my course notes, I have a slightly weaker version of Taylor's Inequality than what we should be using.

Theorem (Taylor's inequality): Suppose that f(x) is n+1 times continuously differentiable on an interval I containing a and let $T_n(x)$ denote he nth Taylor polynomial for f centred at a. If there is a constant $M \geq 0$ such that for any point $x \in I$, we have that $|f^{n+1}(x)| \leq M$, then for any $x \in I$, we have

$$|f(x) - T_n(x)| \le \frac{M}{(n+1)!} |x - a|^{n+1}$$