MATHEMATICS 101 Section 211
Quiz #2, January 23, 2012
Show all your work. Use back of page if necessary. Calculators are not allowed.

Last Name: **First Name:** **UBC Stud. No.:**

Evaluate each of the following. Explain steps as necessary. Full marks *will not* be given to a correct solution without work shown or an explanation given. (2 points each)

1) $\int \frac{4+u^3}{u^4}\,du$

Solution: Breaking up the fraction gives

$$\int \frac{4+u^3}{u^4}\,du = \int \left(\frac{4}{u^4} + \frac{u^3}{u^4}\right) du = \int \frac{4}{u^4} + \int \frac{1}{u}\,du = \frac{-4u^{-3}}{3} + \ln|u| + C$$

Marking Scheme: -0.5 marks if $+C$ is missing. -0.5 marks if $\ln|u|$ is missing (the absolute values). Otherwise full marks for a correct solution.

2) $\int_{-1}^{1} (2x^3 + 5x^7 + 11x^{13} + 17x^{19} + 23x^{29})\,dx$

Solution 1: This is an odd polynomial hence it is an odd function. The interval is symmetric and hence it is 0.

Solution 2: Integrating gives

$$\begin{aligned}
&\int_{-1}^{1} (2x^3 + 5x^7 + 11x^{13} + 17x^{19} + 23x^{29})\,dx \\
&= \left(\frac{2x^4}{4} + \frac{5x^8}{8} + \frac{11x^{14}}{14} + \frac{17x^{20}}{20} + \frac{23x^{30}}{30}\right)\Bigg|_{-1}^{1} \\
&= \frac{2(1)^4}{4} + \frac{5(1)^8}{8} + \frac{11(1)^{14}}{14} + \frac{17(1)^{20}}{20} + \frac{23(1)^{30}}{30} \\
&\quad - \left(\frac{2(-1)^4}{4} + \frac{5(-1)^8}{8} + \frac{11(-1)^{14}}{14} + \frac{17(-1)^{20}}{20} + \frac{23(-1)^{30}}{30}\right) \\
&= 0
\end{aligned}$$

Marking Scheme: -1 mark for a lack of an explanation as to why the function is odd. In solution 2, 1 mark for integral and 1 mark for arithmetic.

3) $\int_{1}^{2} 6x^2\sqrt{8+x^3}\,dx$

Solution 1: We solve this by substitution. Let $u = 8 + x^3$ so $du = 3x^2dx$. Notice that $u(1) = 8 + (1)^3 = 9$ and $u(2) = 8 + (2)^3 = 16$. Hence

$$\int_{1}^{2} 6x^2\sqrt{8+x^3}\,dx = \int_{9}^{16} 2\sqrt{u}\,du = \frac{4u^{\frac{3}{2}}}{3}\Bigg|_{9}^{16} = \frac{4(16)^{\frac{3}{2}}}{3} - \frac{4(9)^{\frac{3}{2}}}{3}$$

$$= \frac{4^4}{3} - \frac{4(27)}{3} = \frac{4(64-27)}{3} = \frac{4(37)}{3} = \frac{148}{3}$$

Solution 2: First we solve the indefinite integral. Let $u = 8 + x^3$ so $du = 3x^2 dx$. Hence

$$\int 6x^2\sqrt{8+x^3}\,dx = \int 2\sqrt{u}\,du = \frac{4u^{\frac{3}{2}}}{3} + C = \frac{4(8+x^3)^{\frac{3}{2}}}{3} + C$$

Now, plug the endpoints in to see

$$\int_1^2 6x^2\sqrt{8+x^3}\,dx = \left(\frac{4(8+x^3)^{\frac{3}{2}}}{3} + C\right)\Bigg|_1^2 = \frac{4(16)^{\frac{3}{2}}}{3} + C - \frac{4(9)^{\frac{3}{2}}}{3} - C$$
$$= \frac{4^4}{3} - \frac{4(27)}{3} = \frac{4(64-27)}{3} = \frac{4(37)}{3} = \frac{148}{3}$$

Marking Scheme: 1.5 marks for correct substitution WITH endpoint changed. 0.5 marks for arithmetic OR 1 mark for indefinite integral and 1 mark for arithmetic.

4) $\displaystyle\int \frac{\cos(\frac{1}{x^2})}{x^3}\,dx$

Solution: We solve this by substitution. Let $u = x^{-2}$ so $du = -2x^{-3}dx$. Hence

$$\int \frac{\cos(\frac{1}{x^2})}{x^3}\,dx = \int \frac{\cos(u)}{-2}\,du = \frac{-\sin(u)}{2} + C = \frac{-\sin(x^{-2})}{2} + C$$

Marking Scheme: 1 mark for correct substitution. One mark for the integral. -0.5 marks if terms are not in terms of x. -0.5 marks for forgetting +C.

5) $\displaystyle\frac{d}{dx}\int_3^{x^2+1} e^{e^t}\,dt$

Solution 1: Using the chain rule, we have

$$\frac{d}{dx}\int_3^{x^2+1} e^{e^t}\,dt = 2xe^{e^{x^2+1}}$$

Solution 2: Let $u = x^2+1$ and $G(u) := \int_3^u e^{e^t}\,dt$. Then, using the chain rule, we have

$$\frac{d}{dx}\int_3^{x^2+1} e^{e^t}\,dt = \frac{dG}{du}\frac{du}{dx} = e^{e^u}(2x) = 2xe^{e^{x^2+1}}$$

Marking Scheme: -1 mark for forgetting chain rule.