MATHEMATICS 101 Section 211
Quiz #2, January 23, 2012
Show all your work. Use back of page if necessary. Calculators are not allowed.

Last Name: First Name: UBC Stud. No.:

Evaluate each of the following. Explain steps as necessary. Full marks will not be given to a correct solution without work shown or an explanation given. (2 points each)

1) \( \int \frac{4 + u^3}{u^4} \, du \)

Solution: Breaking up the fraction gives
\[
\int \frac{4 + u^3}{u^4} \, du = \int \left( \frac{4}{u^4} + \frac{u^3}{u^4} \right) \, du = \int \frac{4}{u^4} \, du + \int \frac{1}{u} \, du = -4u^{-3} + \ln |u| + C
\]

Marking Scheme: -0.5 marks if \(+C\) is missing. -0.5 marks if \(\ln |u|\) is missing (the absolute values). Otherwise full marks for a correct solution.

2) \( \int_{-1}^{1} (2x^3 + 5x^7 + 11x^{13} + 17x^{19} + 23x^{29}) \, dx \)

Solution 1: This is an odd polynomial hence it is an odd function. The interval is symmetric and hence it is 0.

Solution 2: Integrating gives
\[
\int_{-1}^{1} (2x^3 + 5x^7 + 11x^{13} + 17x^{19} + 23x^{29}) \, dx = \left( \frac{2x^4}{4} + \frac{5x^8}{8} + \frac{11x^{14}}{14} + \frac{17x^{20}}{20} + \frac{23x^{30}}{30} \right) \bigg|_{-1}^{1}
\]
\[
= \frac{2(1)^4}{4} + \frac{5(1)^8}{8} + \frac{11(1)^{14}}{14} + \frac{17(1)^{20}}{20} + \frac{23(1)^{30}}{30} - \left( \frac{2(-1)^4}{4} + \frac{5(-1)^8}{8} + \frac{11(-1)^{14}}{14} + \frac{17(-1)^{20}}{20} + \frac{23(-1)^{30}}{30} \right)
\]
\[
= 0
\]

Marking Scheme: -1 mark for a lack of an explanation as to why the function is odd. In solution 2, 1 mark for integral and 1 mark for arithmetic.

3) \( \int_{1}^{2} 6x^2 \sqrt{8 + x^3} \, dx \)

Solution 1: We solve this by substitution. Let \( u = 8 + x^3 \) so \( du = 3x^2 \, dx \). Notice that \( u(1) = 8 + (1)^3 = 9 \) and \( u(2) = 8 + (2)^3 = 16 \). Hence
\[
\int_{1}^{2} 6x^2 \sqrt{8 + x^3} \, dx = \int_{9}^{16} 2\sqrt{u} \, du = \frac{4u^{\frac{3}{2}}}{3} \bigg|_{9}^{16} = \frac{4(16)^{\frac{3}{2}}}{3} - \frac{4(9)^{\frac{3}{2}}}{3}
\]
\[
= \frac{4^4}{3} - \frac{4(27)}{3} = \frac{4(64 - 27)}{3} = \frac{4(37)}{3} = 148
\]
Solution 2: First we solve the indefinite integral. Let \( u = 8 + x^3 \) so \( du = 3x^2 \, dx \). Hence

\[
\int 6x^2 \sqrt{8 + x^3} \, dx = \int 2\sqrt{u} \, du = \frac{4u^{3/2}}{3} + C = \frac{4(8 + x^3)^{3/2}}{3} + C
\]

Now, plug the endpoints in to see

\[
\int_1^2 6x^2 \sqrt{8 + x^3} \, dx = \left[ \frac{4(8 + x^3)^{3/2}}{3} + C \right]_1^2 = \frac{4(16)^{3/2}}{3} + C - \frac{4(9)^{3/2}}{3} - C = \frac{4^4}{3} - \frac{4(27)}{3} = \frac{4(64 - 27)}{3} = \frac{4(37)}{3} = 148
\]

Marking Scheme: 1.5 marks for correct substitution WITH endpoint changed. 0.5 marks for arithmetic OR 1 mark for indefinite integral and 1 mark for arithmetic.

4) \( \int \frac{\cos\left(\frac{1}{x^2}\right)}{x^3} \, dx \)

Solution: We solve this by substitution. Let \( u = x^{-2} \) so \( du = -2x^{-3} \, dx \). Hence

\[
\int \frac{\cos\left(\frac{1}{x^2}\right)}{x^3} \, dx = \int \frac{\cos(u)}{-2} \, du = \frac{-\sin(u)}{2} + C = \frac{-\sin(x^{-2})}{2} + C
\]

Marking Scheme: 1 mark for correct substitution. One mark for the integral. -0.5 marks if terms are not in terms of x. -0.5 marks for forgetting +C.

5) \( \frac{d}{dx} \int_3^{x^2 + 1} e^t \, dt \)

Solution 1: Using the chain rule, we have

\[
\frac{d}{dx} \int_3^{x^2 + 1} e^t \, dt = 2xe^{x^2 + 1}
\]

Solution 2: Let \( u = x^2 + 1 \) and \( G(u) := \int_3^u e^t \, dt \). Then, using the chain rule, we have

\[
\frac{d}{dx} \int_3^{x^2 + 1} e^t \, dt = \frac{dG}{du} \frac{du}{dx} = e^u (2x) = 2xe^{x^2 + 1}
\]

Marking Scheme: -1 mark for forgetting chain rule.