Midterm Practice Problems

Disclaimer: I in no way suggest that this is comprehensive. Students should review their notes, assignments and suggested problems in conjunction with these practice problems. Another helpful hint is to look at quizzes of other sections. Sample midterms are on the common page.

For reference, you may need some or all of the following formulas

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2} \qquad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} \qquad \sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$$

1) Write the following in terms of the summation notation then evaluate.
   i. The sum of the first 10 numbers
   ii. The sum of two five hundred times
   iii. The sum of the first 155 even numbers
   iv. The sum of the first 155 odd numbers (Hint for evaluating: use the previous problem)
   v. The sum of the the numbers between 90 and 100 inclusive.
   vi. The sum of the first 100 numbers in the sequence 6,12,18,24,30,...

2) Evaluate the following
   i. $\sum_{i=2}^{5}(4+\sqrt{9^i})$
   ii. $\sum_{i=0}^{n}(5+3i)$
   iii. $\sum_{i=0}^{200}(i-3)^2$
   iv. $\sum_{i=15}^{n} i^3+2i$
   v. $\sum_{i=1}^{n} \frac{1}{i(i+1)}$

3) Using the definition of an integral (with right endpoints in the Riemann sums), evaluate
   i. $\int_0^4 3dx$
   ii. $\int_0^1 (2x+3)dx$
   iii. $\int_{-3}^2 (x-5)dx$
   iv. $\int_{-1}^1 (x^2-1)dx$
   v. $\int_3^6 (x+1)^3 dx$
   vi. $\int_2^4 (x^2+2x+1)dx$
   vii. $\int_0^2 e^{2x}dx$

4) Write the following as a definite integral
   i. $\lim_{n\to\infty} \sum_{i=1}^{n} \left(5+\frac{3i}{n}\right)^4 \left(\frac{2}{n}\right)$
   ii. $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{i}{n(n+i)}$
   iii. $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{\cos(6+\frac{3i}{n})^2}{2n+i}$

5) Using the left endpoint, right endpoint, and midpoint rules, give an approximation of the area under the curve for each of the following functions from 0 to 2 with 4 subintervals

i. $f(x) = 3$

ii. $f(x) = 4x + 1$

iii. $f(x) = x^2 - 4$

6) Using high school geometry, evaluate the following

i. $\int_0^3 \sqrt{9 - x^2} dx$

ii. $\int_0^3 (\sqrt{9 - x^2} + 1) dx$

iii. $\int_{-2}^2 |x| dx$

iv. $\int_{-2}^2 |x - 1| dx$

v. $\int_{-2}^2 f(x) dx$ where $f(x) = \begin{cases} 2x + 2 & \text{for } -2 \leq x \leq 0 \\ 3\sqrt{4 - x^2} & \text{for } 0 \leq x \leq 2 \end{cases}$

7) Prove that $0.367 \leq \int_0^1 e^{-x^2} dx \leq 1$.

8) Prove that $\int_0^{\frac{\pi}{4}} \sin(x^2) \leq 1 - \frac{1}{\sqrt{2}}$ (Midterm 2010 - Final problem).

9) If $\int_1^5 f(x) dx = 20$ and $\int_4^5 f(x) dx = 3$, find $\int_1^4 f(x) dx$.

10) State the fundamental theorem of calculus part I.

11) Compute the following

i. $\frac{d}{dx} \int_0^x \sin(\cos(\sin(t))) dt$.

ii. $\frac{d}{dx} \int_2^x e^{\cos(t)} dt$.

iii. $\frac{d}{dx} \int_3^x e^{\cos(t)} dt$.

iv. $\frac{d}{dx} \int_x^2 e^{\cos(t)} dt$.

v. $\frac{d}{dx} \int_2^{x^2+x+1} e^{\cos(t)} dt$.

vi. $\frac{d}{dx} \int_{x^2-1}^{x^2+1} e^{\cos(t)} dt$.

12) State the fundamental theorem of calculus part II.

13) Find all antiderivatives of $f(x) = x^2 + 1$.

14) Evaluate the following (You'll need all the techniques you know to solve these). Verify your answers in the indefinite cases by differentiating.

i. $\int_1^2 \frac{1+u^2}{u^3} du$

ii. $\int (u^2 - 1)^2 du$

iii. $\int \arctan(x) dx$

iv. $\int \tan(x) dx$

v. $\int_0^{\frac{\pi}{4}} \sec^2(t) dt$

vi. $\int e^u \sin(u) du$

vii. $\int \frac{\ln(x)}{x^2} dx$

viii. $\int_1^2 \frac{e^{x^{-1}}}{x^2} dx$

ix. $\int \frac{1}{2x} dx$

x. $\int \frac{1}{\sqrt{1-x^2}} dx$

xi. $\int_0^{\pi} \sec(x) \tan(x) dx$

xii. $\int 10^u du$

xiii. $\int_{-1}^{1} x^2 \sin(x) dx$

xiv. $\int_0^1 \frac{4}{t^2+1} dt$

xv. $\int \sec(t) dt$

xvi. $\int_1^4 \frac{1}{x^3} dx$

xvii. $\int \frac{t^9}{1+t^{20}} dt$

xviii. $\int x \ln(x) dx$

xix. $\int_{-1}^{1} |x^3 - x| dx$

xx. $\int \frac{x^2}{1+x^2} dx$

xxi. $\int_0^{\pi} \sin(x) dx$

xxii. $\int \frac{3}{t \ln(t)} dt$

xxiii. $\int_0^{\pi} \cos(x) dx$

xxiv. $\int x^2 e^x dx$

xxv. $\int \frac{4x^3+1}{x^4+x} dx$ (Midterm 2008,2010)

xxvi. $\int 4x^2 \sqrt{x^3+2} dx$

xxvii. $\int \ln(t) dt$

xxviii. $\int_{-5}^{3} |x^2 - 4| dx$

xxix. $\int 2t \log_3(t) dt$

xxx. $\int_{-10}^{10} (2x^3 + 5x^7 + 11x^{13} + 17x^{19} + 23x^{29}) dx$

xxxi. $\int 6 dt$

xxxii. $\int_0^{\ln(3)} \frac{e^t}{e^t+1} dt$

xxxiii. $\int x^3 \cos(4x^4 + 10) dx$

xxxiv. $\int_{-1}^{3} (x^2 + x + 1) dx$

xxxv. $\int \frac{x}{1+x^4} dx$

xxxvi. $\int \sec(t) \tan(t) dt$

xxxvii. $\int x \sin(x) dx$

xxxviii. $\int_{e^2}^{e^3} \ln(x)^2 dx$

xxxix. $\int u \arctan(u) du$

xl. $\int \frac{1+x}{1+x^2} dx$

xli. $\int x e^{-x} dx$

xlii. $\int \cos(x) e^{\sin(x)} dx$

xliii. $\int \cos(x) \sin(x) dx$

15) A particle moves along a line so its velocity is $v(t) = t^2 - 2t - 5$. Find its displacement and distance traveled during the time period $0 \leq t \leq 10$.

16) The linear density of a rod of length four metres is given by $\rho(x) = 9 + 2\sqrt{x}$ measured in kilograms per meter where x is measured in metres from one end of the rod. Find the total mass of the rod.

17) Water flows from the bottom of a storage tank at a rate of $r(t) = |200 - 4t|$ litres per minute where $0 \leq t \leq 50$. Find the amount of water that flows from the tank during the first 10 minutes.

18) Find the area of the region bounded by the curves in each question

i. $y = \sin(x)$, $y = \cos(x)$, $x = 0$, $x = \frac{\pi}{2}$.

ii. $y = x^2$, $x = y^2$

iii. $y = x^3 - x$, $y = 3x$

iv. $y = x^2$, $y = x^4$

v. $y^2 - 1 = x$, $x = y + 3$

vi. $y = |x|$, $y = x^2 - 2$

19) Find a number b so that the line $y = b$ divides the region bounded by the curves $y = x^2$ and $y = 4$ into two regions of equal area.

20) Using the washer method (or the bowl method), compute the volume of revolutions of the following around the x-axis

i. A sphere with radius r

ii. A cone with radius r and height h

iii. $y = e^x$, $y = 0$, $x = 0$, $x = 1$

iv. $y = x^2$, $y^2 = x$

v. $y = \sqrt{x - 1}$, $x = 2$, $x = 5$, $y = 0$

21) Using the washer method (or the bowl method), compute the volume of revolutions of the following around the y-axis

i. $y^2 = x$, $x = 2y$

ii. $x = y - y^2$, $x = 0$

iii. $y = x^{\frac{2}{3}}$, $x = 1$, $y = 0$.

22) Using the washer method, compute the volume of revolutions of the following around the specified line

i. $y = x^4$, $y = 1$ around the line $y = 2$.

ii. $x = y^2$, $x = 1$ around the line $x = 1$

iii. $y = x^2$, $x = y^2$ around the line $y = -1$ then the same problem around the line $x = -1$

23) Using cross sectional areas, compute the following volumes

i. A sphere with radius r

ii. A cone with radius r and height h

iii. A tetrahedron with base an equilateral triangle and height h

iv. A pyramid with a square base and height h.

v. The cap of a sphere of radius r and height of the cap h.

vi. A shape with base a circle of radius r and parallel cross sections perpendicular to the base are squares.

24) Suppose that 2 joules of work is needed to stretch a spring from its natural length of 30cm to a length of 42cm. How much work is needed to stretch it from 35cm to 40cm? How far beyond its natural length will a force of 30N keep the spring stretched?

25) If the work required to stretch a spring 1ft beyond its natural length is 24ft-lb, how much work is needed to stretch it 0.75 ft beyond its natural length?

26) A chain lying on the ground is 10m long and its mass is 80kg. How much work is required to raise one end f the chain to a height of 6m?

27) How much work is done by a weightlifter in raising a 90kg barbell form the floor to a height of 2m?

28) A 15ft chain weighs 40 lb. and hangs from a ceiling. Find the work done in lifting the lower end of the chain to the ceiling so that it's level with the upper end.

29) A hemispherical tank of radius 5 is filled to the top with water. Find the amount of work needed to empty the tank (density of water is $1000kg/m^3$).

30) A cylindrical tank of radius 2m and height 4m is positioned with its circular base on the ground. Water is extracted from the tank via an outlet that sticks out of the tank an additional 1m. Find the total work done to empty the tank if it is initially half full.

31) Find the average value of the following on the given interval

   i. $f(x) = x^2$ on $[-2, 2]$
   ii. $f(x) = x^{-2}$ on $[1, 2]$
   iii. $f(x) = \sin(x)$ on $[0, \pi]$
   iv. $f(x) = \cos(x)$ on $[0, \pi]$
   v. $f(x) = xe^{-x^2}$ on $[0, 3]$

32) Find all values of b such that the average value of $f(x) = 2_6x - 3x^2$ on the interval $[0, b]$ is equal to 3.

33) The linear density in a rod 8m long is $\frac{12}{\sqrt{x+1}}$kg/m where x is measured in metres from one end of the rod. Find the average density of the rod.

34) In a certain city, the temperature in Fahrenheit after 9am was modeled by the function $T(t) = 50 + 14\sin(\frac{\pi t}{12})$. Find the average temperature during the period from 9am to 9pm.

35) Find a constant c such that $f_{av} = f(c)$ where $f(x) = (x-2)^2$ on $[2, 5]$.