Integration Bee

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Question 1

Compute $\int_0^\pi \sin(x) dx$.

Question 1 - Solution

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$$\int_0^\pi \sin(x)dx = -\cos(x)|_0^\pi$$

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$$\begin{aligned}\int_0^\pi \sin(x)dx &= -\cos(x)|_0^\pi \\ &= -\cos(\pi) - (-\cos(0)) \\ &= -(-1) - (-1) \\ &= 2\end{aligned}$$

Question 2

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Substituting gives

$$\begin{aligned} \int e^{\sqrt{x}} dx &= \int 2ue^{u} du \\ &= 2 \int ue^{u} du \end{aligned}$$

Question 2 - Solution (Continued)

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&= 2\sqrt{u}\Big|_1^2 \\
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&= 2\sqrt{2} - 2 \\
&= 0.82842712474619009760337\ldots
\end{aligned}$$

Question 4

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&= -x^{11} \cos(x) \sin(x) \tan(x) \\
&= -f(x)
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$$\begin{aligned} \int \ln(x)dx &= x\ln(x) - \int dx \\ &= x\ln(x) - x \end{aligned}$$