Error Checking Codes

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In the beginning...

We had very primitive methods of long distance communication...

Smoke Signals

String Phone

Telephone

Wired Internet

Wireless Internet

Transmitting Messages

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What has changed over time is what is causing the error but nonetheless errors occur in communications.

A Problem

- Suppose you wanted to send a message "Call me at 519-888-4567".
- If the recipient receives the message "**R**all me **t**t 519-888-4567", the recipient might be annoyed but could probably determine the message.
- However, if the recipient receives the message "Call me at 519-888-45**2**7", then we have a big issue on our hands and all we did was change one number.

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We want to send messages without any mistakes using error prone devices that can be influenced by very tiny external factors.

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Samples of sources of errors:

- CDs and DVDs can have scratches
- Wifi has multiple competing and interferring signals.
- Data on a hard drive is largely influenced by magnetic interruptions.
- Optical fibre wires suffer from noise, cable breaks, etc.

Attempt 1: Repetition

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Message: "Transfer $3592.15 into account number 123456789"

Transferred Messages:

- "Transfer $3**3**92.15 into account number 123456789"
- "Tra**m**sfer $359**3**.15 into account number 1234**3**6789"
- "Tr**G**nsfer $3592.15 into acc**l**unt number 12**9**4567**4**9"
- "Transfer $359**8**.15 into account number 123456**8**89"
- "Transfer $3592.15 into account number 123456789"

Here the repeated trick worked and the message received would be the most common message in each character position, which in this case is the original message.

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- "Transfer $3592.15 into account number 123456789"

Here the repeated trick method fails and the message received would be"Transfer $3592.15 into account number 1234**3**6789" even though we did mange to get the message sent error free one time above! This might not even be that contrived of a situation if the data is being send across some wire with damage in a specific spot always causing a certain bit to change.

Pros and Cons

What are some pros and cons to this method?

Attempt 1: Repetition Summary

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- Detects and corrects errors often.

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- Provides at least some improved chance of working.
- Detects and corrects errors often.

Cons:

- How many times do we send the message? Lots of extra overhead each time we send.
- Don't know when we actually receive the correct message.
- If there is a specific error that always causes a specific bit to differ, this method will never work.

This is actually used in practice! (usually for storage but not for transmission).

Attempt 2: Redundancy

A possibly source of the previous problem is that looking at the individual numbers themselves have far too much importance. Why don't we lengthen each digit and send the important data redundantly?

One way to do this is to use words to describe the numbers.

Attempt 2: Redundancy Redundancy

Message to send: “123456789”

Message we send: “one two three four five six seven eight nine”

Message received: “pne tto thrxp gour eive sik teven night pwne”

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Even with 11 errors out of 36 non blank characters, we could still read the message.

In practice this isn't the exact method used but gives the general idea for Hamming Codes (we'll discuss these later!).

Humans and computers can decipher this

For emaxlpe, it deson?t mttaer in waht oredr the ltteers in a wrod aepapr, the olny iprmoatnt tihng is taht the frist and lsat ltteer are in the rghit pcale. The rset can be a toatl mses and you can sitll raed it wouthit pobelrm.

S1M1L4RLY, YOUR M1ND 15 R34D1NG 7H15 4U70M471C4LLY W17HOU7 3V3N 7H1NK1NG 4BOU7 17.

http://www.livescience.com/18392-reading-jumbled-words.html

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Cons:

- How should we encode the data? What redundancy should we used that's optimal?
- If there is a specific error that always causes a specific bit to differ, this method will never work.

These techniques when modified are used in practice. (We'll see examples later)

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- Let's use an example in binary (using only zeroes and ones). Suppose we wanted to send the message 01100110110.
- This message has 11 digits and 6 of them are ones.
- Suppose we changed exactly one digit. Then the number of ones would have changed to either 5 or 7 depending on if we changed a 1 to a 0 or a 0 to a 1.

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  1 if the number of ones is odd and 0 if the number of ones is even.

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- Here, since we have 6 ones, we will add an extra 0 to the end of the checksum. Thus, we send the message

  011001101100

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then we would know the message is incorrect since the check sum digit says there should be an even number of ones but there is an odd number of ones and this is an error.

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- Even though we know there is an error, our scheme cannot determine where the error is. It would just reject the message.
- We call such a message potentially valid if it passes our checksum test.

Attempt 3: Checksum

Question: Which of the following are potentially valid messages?

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2. 11101110101
3. 000000000000
4. 1110110101001

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However, despite it's primacy, this type of idea is widely used. In what follows we give two examples of this idea being used.

Idea 1: Luhn's Algorithm

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- In our everyday life, we tend to deal more with decimal digits than with binary digits.
- Is there an algorithm that works for decimal numbers and is used?
- Yes! An algorithm due to Luhn is widely used and many people in this room have something that has this algorithm embedded in it.

My Credit Card Number

- Take for example my credit card number:

  4025901376813517

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- First, for all the digits in even positions, add them up. Call this number E.

$$E = 0 + 5 + 0 + 3 + 6 + 1 + 5 + 7 = 27$$

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Credit Card Number: 4025901376813517 $E = 27$

- Then, for all the digits in odd positions, double the digits then add all the numbers. Call this number O.
- The odd positioned digits are $4, 2, 9, 1, 7, 8, 3, 1$.

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- Then, for all the digits in odd positions, double the digits then add all the numbers. Call this number O.
- The odd positioned digits are $4, 2, 9, 1, 7, 8, 3, 1$.
- Doubling gives $8, 4, 18, 2, 14, 16, 6, 2$.

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- Adding the digits together:

$$O = 8 + 4 + 1 + 8 + 2 + 1 + 4 + 1 + 6 + 6 + 2 = 43$$

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- Lastly, add E and O. If the sum is divisible by 10, we call the number potentially valid.

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- The odd positioned digits are $4, 2, 9, 1, 7, 8, 3, 1$.
- Doubling gives $8, 4, 18, 2, 14, 16, 6, 2$.
- Adding the digits together:

$$O = 8 + 4 + 1 + 8 + 2 + 1 + 4 + 1 + 6 + 6 + 2 = 43$$

- Lastly, add E and O. If the sum is divisible by 10, we call the number potentially valid.
- Notice that

$$E + O = 70$$

and hence my credit card number is potentially valid.

Which of these are potentially valid?

1. 9054328746541532
2. 6154328746541537
3. 4142658749512432
4. 9974865142354173

Solutions

9054328746541532

Solutions

9054328746541532

Even Positions

- $0, 4, 2, 7, 6, 4, 5, 2$
- Sum is

$$0+4+2+7+6+4+5+2 = 30.$$

Odd Positions

- $9, 5, 3, 8, 4, 5, 1, 3$
- Doubling:
  $18, 10, 6, 16, 8, 10, 2, 6$
- Sum of digits is

$$1+8+1+0+6+1+6 \\ +8+1+0+2+6 = 40.$$

Sum is $30 + 40 = 70$ which is divisible by 10.

The number is potentially valid.

Solutions

6154328746541537

Solutions

6154328746541537

Even Positions

- $1, 4, 2, 7, 6, 4, 5, 7$
- Sum is

$$1+4+2+7+6+4+5+7 = 36.$$

Odd Positions

- $6, 5, 3, 8, 4, 5, 1, 3$
- Doubling:
  $12, 10, 6, 16, 8, 10, 2, 6$
- Sum of digits is

$$1+2+1+0+6+1+6 \\ +8+1+0+2+6 = 34.$$

Sum is $36 + 34 = 70$ which is divisible by 10.

The number is potentially valid.

Solutions

4142658749512432

Solutions

4142658749512432

Even Positions

- $1, 2, 5, 7, 9, 1, 4, 2$
- Sum is

$$1+2+5+7+9+1+4+2 = 31.$$

Odd Positions

- $4, 4, 6, 8, 4, 5, 2, 3$
- Doubling:
  $8, 8, 12, 16, 8, 10, 4, 6$
- Sum of digits is

$$8+8+1+2+1+6+8 \\ +1+0+4+6 = 45.$$

Sum is $31 + 45 = 76$ which is not divisible by 10.

The number is not potentially valid (invalid).

Solutions

9974865142354173

Even Positions

- $9, 4, 6, 1, 2, 5, 1, 3$
- Sum is

$$9+4+6+1+2+5+1+3 = 31.$$

Odd Positions

- $9, 7, 8, 5, 4, 3, 4, 7$
- Doubling:
  $18, 14, 16, 10, 8, 6, 8, 14$
- Sum of digits is

$$\begin{aligned} &1+8+1+4+1+6 \\ &+1+0+0+8+6 \\ &+8+1+4 = 49. \end{aligned}$$

Sum is $31 + 49 = 80$ which is divisible by 10.

The number is potentially valid.

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- Luhn's Algorithm can detect the two most common data entry errors for numbers:
  1. Getting a single digit incorrect.
  2. Switching adjacent digits (with the exception of switching a 9 and 0).

Error Detection in Luhn's Algorithm (single digit).

Actual: 9054328746541532 Received: 9054338746541532

Error Detection in Luhn's Algorithm (single digit).

Actual: 9054328746541532 Received: 9054338746541532

Even Positions

- $0, 4, 3, 7, 6, 4, 5, 2$
- Sum is

$$0+4+3+7+6+4+5+2 = 31.$$

Odd Positions

- $9, 5, 3, 8, 4, 5, 1, 3$
- Doubling:
  $18, 10, 6, 16, 8, 10, 2, 6$
- Sum of digits is

$$1+8+1+0+6+1+6 \\ +8+1+0+2+6 = 40.$$

Sum is $31 + 40 = 71$ which is not divisible by 10.

The number is not potentially valid.

Error Detection in Luhn's Algorithm (single digit).

Actual: 9054328746541532 Received: 9054327846541532

Error Detection in Luhn's Algorithm (single digit).

Actual: 9054328746541532 Received: 9054327846541532

Even Positions

- $0, 4, 2, 8, 6, 4, 5, 2$
- Sum is

$$0+4+2+8+6+4+5+2 = 31.$$

Odd Positions

- $9, 5, 3, 7, 4, 5, 1, 3$
- Doubling:
  $18, 10, 6, 14, 8, 10, 2, 6$
- Sum of digits is

$$1+8+1+0+6+1+4$$
$$+8+1+0+2+6 = 38.$$

Sum is $31 + 38 = 69$ which is not divisible by 10.

The number is not potentially valid.

Error Detection in Luhn's Algorithm (Does not detect the 90 swap).

Actual: 9054328746541532 Received: 0954328746541532

Error Detection in Luhn's Algorithm (Does not detect the 90 swap).

Actual: 9054328746541532 Received: 0954328746541532

Even Positions

- $9, 4, 2, 7, 6, 4, 5, 2$
- Sum is

$$9+4+2+7+6+4+5+2 = 39.$$

Odd Positions

- $0, 5, 3, 8, 4, 5, 1, 3$
- Doubling:
  $0, 10, 6, 16, 8, 10, 2, 6$
- Sum of digits is

$$0 + 1 + 0 + 6 + 1 + 6 + 8 + 1 + 0 + 2 + 6 = 31.$$

Sum is $39 + 31 = 70$ which is divisible by 10.

The number is potentially valid.

Luhn's Algorithm Sanity Check

Which of these numbers are potentially valid given that your credit card number is:

1234567899000123

1. 1234657899000123
2. 1234597899000123
3. 1234567890900123
4. 3214567899000123
5. 2134567899000213

Luhn's Algorithm Sanity Check

Which of these numbers are potentially valid given that your credit card number is:

1234567899000123

1. 1234657899000123 not potentially valid
2. 1234597899000123 not potentially valid
3. 1234567890900123 potentially valid
4. 3214567899000123 potentially valid
5. 2134567899000213 ???

Error Detection in Luhn's Algorithm (Does not detect the 90 swap).

2134567899000213

Even Positions

- $1, 4, 6, 8, 9, 0, 2, 3$
- Sum is

$$1+4+6+8+9+0+2+3 = 33.$$

Odd Positions

- $2, 3, 5, 7, 9, 0, 0, 1$
- Doubling:
  $4, 6, 10, 14, 18, 0, 0, 2$
- Sum of digits is

$$4 + 6 + 1 + 0 + 1 + 4 + 1 + 8 + 0 + 0 + 2 = 27.$$

Sum is $33 + 27 = 60$ which is divisible by 10.

The number is potentially valid.

Pros and Cons

What are some pros and cons to this method?

Summary of Luhn's Algorithm

Pros:

- Detects most common single errors.

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Pros:

- Detects most common single errors.

Cons:

- Cannot detect multiple errors.
- Cannot correct errors, only detect.

Idea 2: $(7, 4)$ Hamming Code

The $(7, 4)$ Hamming Code is a redundancy error checking code that takes in four bits and produces a 7 bit encoding consisting of the original four bits and three parity bits $d_1 d_2 d_3 d_4 p_1 p_2 p_3$.

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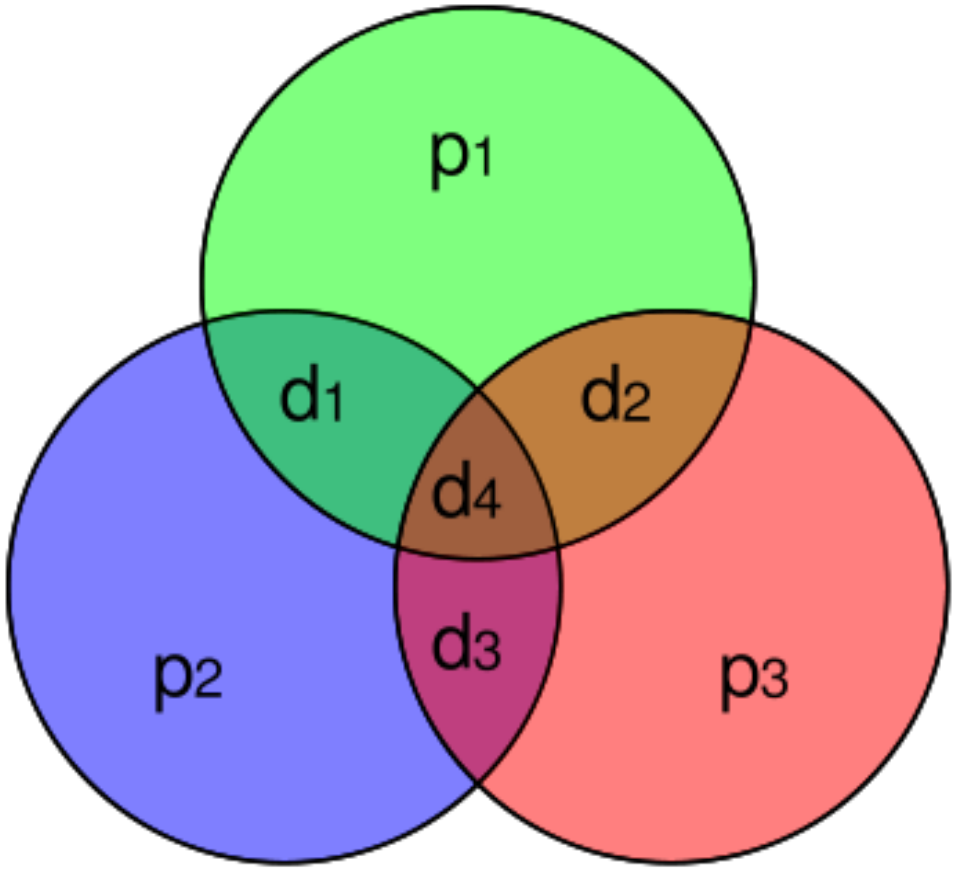
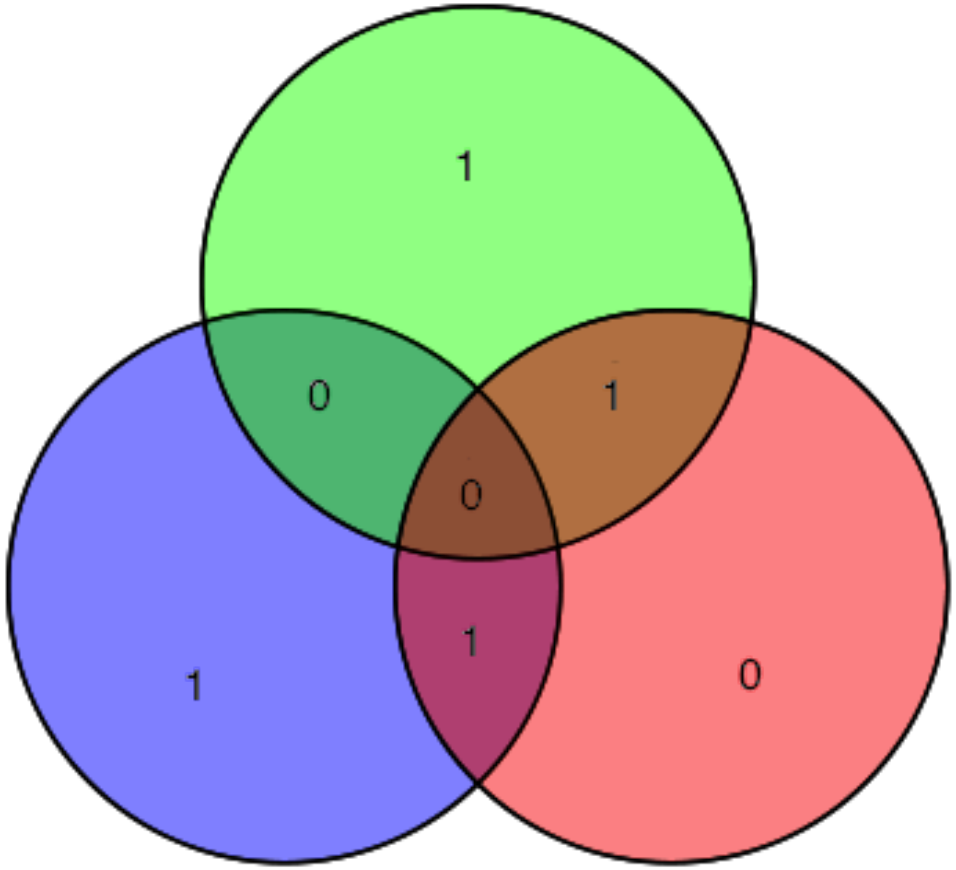


Note: Ordering of digits differs in practice.

$(7, 4)$ Hamming Code (Courtesy Wikipedia)



Your Turn! $(7, 4)$ Hamming Code

Complete the table below

Original	Encoding	Original	Encoding
0000	0000000	1000	1000110
0001	0000111	1001	1001001
0010	0000011	1010	1010101
0011	0000100	1011	1011010
0100	0100101	1100	
0101	0101010	1101	
0110	0110110	1110	
0111	0111001	1111	

$(7, 4)$ Hamming Code

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0000	0000000	1000	1000110
0001	0000111	1001	1001001
0010	0000011	1010	1010101
0011	0000100	1011	1011010
0100	0100101	1100	1100011
0101	0101010	1101	1101100
0110	0110110	1110	1110000
0111	0111001	1111	1111111

$(7, 4)$ Hamming Code Error Correction

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- Out parity bits tells us there is a problem. In digits 1, 3 and 4, there is a single one and so the second parity bit should be a 1. However it is a 0.

(7, 4) Hamming Code Error Correction

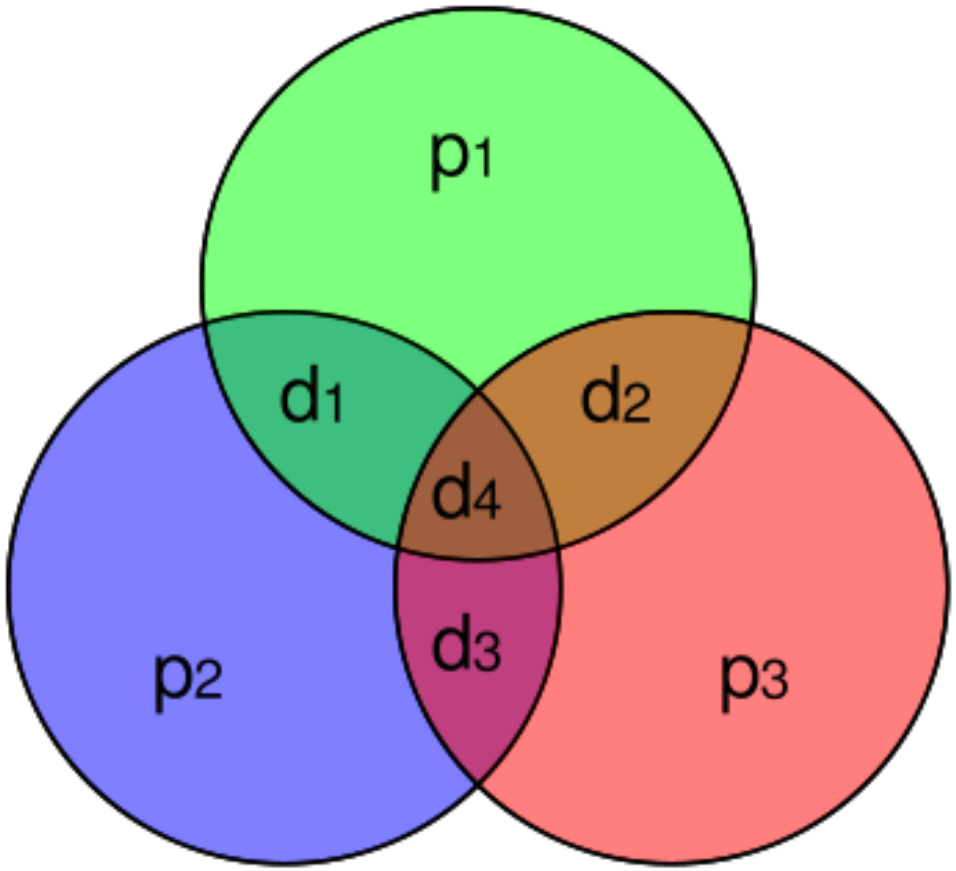
With this Hamming Code, we can actually detect and correct single errors!

- Suppose we receive the message 0110101.
- Out parity bits tells us there is a problem. In digits 1, 3 and 4, there is a single one and so the second parity bit should be a 1. However it is a 0.
- Continuing our check:
  - Digits 1,2,4 contain a single 1 so the first parity bit should be a 1.
  - Digits 2,3,4 contain 2 copies of 1 so the last parity bit should be a 0 but it is a 1.
- So the two check sums containing the digit 3 are incorrect. This means that digit 3 is incorrect and changing the 1 back to a 0 recovers the message.
- This assumes that there was only one message in the code!

(7, 4) Hamming Code Error Correction

The following have a single error. Find and correct the error

1. 0010110
2. 1000001
3. 1011011



$(7,4)$ Hamming Code Error Correction

The following have a single error. Find and correct the error

1. 0110110
2. 1001001
3. 1011010

Pros and Cons

What are some pros and cons to this method?

$(7, 4)$ Hamming Code

Pros:

- Detects and corrects single errors.

$(7, 4)$ Hamming Code

Pros:

- Detects and corrects single errors.

Cons:

- Might erroneously correct errors if multiple errors are made.
- Can detect errors up to 3 bits incorrect (but lose ability to correct single errors).

This is actually used in practice! For example DRAM still uses this. This was originally created by Richard Hamming while working for Bell Telephone Laboratories. He was frustrated by the error-prone punch card readers and wanted to create a system to double check the code.

Summary

- Error checking codes are vital to many current systems.
- Many complicated codes exist to help solve this problem. (Reed-Solomon Codes, QR Decompositions etc.)
- Error checking codes exist in many current objects like CDs, credit cards and SIN numbers.

Thank you!

Thank you!

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