

Abbreviated version of the reversing an array (special thanks to Collin Roberts and Jonathan Buss for this code.)  
 Notice that we have suppressed the requirement that ( $n > 0$ ) throughout. We could include it but choose not to in the interest of space.

Let  $\text{Inv}'(j)$  be the formula

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$$\left( \forall x \left( (((1 \leq x) \wedge (x \leq n)) \rightarrow (R[x] = r_x)) \right) \right)$$


$$\wedge \left( \left( j \leq x \right) \wedge \left( x \leq \frac{n+1}{2} \right) \rightarrow ((R[x] = r_x) \wedge (R[n+1-x] = r_{n+1-x})) \right) \right).$$



$$\{ \left( \forall x \left( ((1 \leq x) \wedge (x \leq n)) \rightarrow (R[x] = r_x)) \right) \right) \} \quad \text{Implied(a)}$$


$$\{ \left( \text{Inv}'(1) \wedge (1 \leq (\frac{n}{2} + 1)) \right) \}$$


$$j = 1 ; \quad \text{Assignment}$$


$$\{ \left( \text{Inv}'(j) \wedge (j \leq (\frac{n}{2} + 1)) \right) \}$$


$$\text{while } 2 * j <= n \{ \quad \text{Assignment}$$


$$\{ \left( (\text{Inv}'(j) \wedge (j \leq (\frac{n}{2} + 1))) \wedge ((2 \cdot j) < n) \right) \} \quad \text{Partial-While}$$


$$\{ \left( \text{Inv}'((j+1))[R'/R] \wedge ((j+1) \leq (\frac{n}{2} + 1)) \right) \} \quad \text{Implied(c)}$$


$$t = R[j] ; \quad R[j] = R[n+1-j] ; \quad R[n+1-j] = t ; \quad \text{Lemma}$$


$$\{ \left( \text{Inv}'((j+1)) \wedge ((j+1) \leq (\frac{n}{2} + 1)) \right) \}$$


$$j = j + 1 ; \quad \text{Assignment}$$


$$\{ \left( \text{Inv}'(j) \wedge (j \leq (\frac{n}{2} + 1)) \right) \} \}$$


$$\{ \left( (\text{Inv}'(j) \wedge (j \leq (\frac{n}{2} + 1))) \wedge ((2 \cdot j) > n) \right) \} \quad \text{Partial-While}$$


$$\{ \left( \forall x \left( ((1 \leq x) \wedge (x \leq n)) \rightarrow (R[x] = r_{n+1-x})) \right) \} \quad \text{Implied(b)}$$


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On the back side of this document is this example in full detail (special thanks to Collin Roberts and Jonathan Buss for the original code). Due to page width restrictions, let

- $R' = R\{j \leftarrow R[((n+1)-j)]\}\{((n+1)-j) \leftarrow R[j]\}$
- $R'' = R\{j \leftarrow R[((n+1)-j)]\}\{((n+1)-j) \leftarrow t\}$
- $R''' = R\{((n+1)-j) \leftarrow t\}$



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$$\begin{array}{l}
\{\forall x (((1 \leq x) \wedge (x \leq n)) \rightarrow (R[x] = r_x))\} \dagger \\
\{\text{Inv}'(1) \wedge (1 \leq (\frac{n}{2} + 1))\} \dagger \\
j = 1; \\
\{\left(\forall x \left(((1 \leq x) \wedge (x < j)) \rightarrow ((R[x] = r_{n+1-x}) \wedge (R[n+1-x] = r_x))\right)\right. \\
\quad \wedge \left(((j \leq x) \wedge (x \leq \frac{n+1}{2})) \rightarrow ((R[x] = r_x) \wedge (R[n+1-x] = r_{n+1-x}))\right)\} \dagger \\
\quad \text{Assignment} \\
\quad \text{while } (2 * j <= n) \{ \\
\quad \quad \{\left(\left(\forall x \left(((1 \leq x) \wedge (x < j)) \rightarrow ((R[x] = r_{n+1-x}) \wedge (R[n+1-x] = r_x))\right)\right. \\
\quad \quad \quad \wedge \left(\left((j \leq x) \wedge (x \leq \frac{n+1}{2})\right) \rightarrow ((R[x] = r_x) \wedge (R[n+1-x] = r_{n+1-x}))\right)\}\} \wedge \left((j \leq \frac{n}{2} \wedge ((2 \cdot j) <= n))\right) \dagger \\
\quad \quad \text{Partial-While} \\
\quad \quad \{\left(\left(\forall x \left(((1 \leq x) \wedge (x < (j+1))) \rightarrow ((R'[x] = r_{n+1-x}) \wedge (R'[n+1-x] = r_x))\right)\right. \\
\quad \quad \quad \wedge \left(\left((j \leq x) \wedge (x < (j+1))\right) \rightarrow ((R'[x] = r_{n+1-x}) \wedge (R'[n+1-x] = r_x))\right)\}\} \dagger \\
\quad \quad \text{Implied}(c) \\
\quad \quad \wedge \left(\left(((j+1) \leq x) \wedge (x \leq \frac{n+1}{2})\right) \rightarrow ((R'[x] = r_x) \wedge (R'[n+1-x] = r_{n+1-x}))\right)\} \wedge ((j+1) \leq (\frac{n}{2} + 1)) \dagger \\
t = R[j]; \\
\quad \{\left(\left(\forall x \left(((1 \leq x) \wedge (x < (j+1))) \rightarrow ((R''[x] = r_{n+1-x}) \wedge (R''[n+1-x] = r_x))\right)\right. \\
\quad \quad \wedge \left(\left(((j+1) \leq x) \wedge (x \leq \frac{n+1}{2})\right) \rightarrow ((R''[x] = r_x) \wedge (R''[n+1-x] = r_{n+1-x}))\right)\}\} \wedge ((j+1) \leq (\frac{n}{2} + 1)) \dagger \\
R[j] = R[n+1-j]; \\
\quad \{\left(\left(\forall x \left(((1 \leq x) \wedge (x < (j+1))) \rightarrow ((R'''[x] = r_{n+1-x}) \wedge (R'''[n+1-x] = r_x))\right)\right. \\
\quad \quad \wedge \left(\left(((j+1) \leq x) \wedge (x \leq \frac{n+1}{2})\right) \rightarrow ((R'''[x] = r_x) \wedge (R'''[n+1-x] = r_{n+1-x}))\right)\}\} \wedge ((j+1) \leq (\frac{n}{2} + 1)) \dagger \\
R[n+1-j] = t; \\
\quad \{\left(\left(\forall x \left(((1 \leq x) \wedge (x < (j+1))) \rightarrow ((R[x] = r_x) \wedge (R[n+1-x] = r_x))\right)\right. \\
\quad \quad \wedge \left(\left(((j+1) \leq x) \wedge (x \leq \frac{n+1}{2})\right) \rightarrow ((R''[x] = r_x) \wedge (R''[n+1-x] = r_{n+1-x}))\right)\}\} \wedge ((j+1) \leq (\frac{n}{2} + 1)) \dagger \\
\quad \text{Assignment} \\
\quad \wedge \left(\left(((j+1) \leq x) \wedge (x < (j+1))\right) \rightarrow ((R[x] = r_x) \wedge (R[n+1-x] = r_x))\right)\} \wedge ((j+1) \leq (\frac{n}{2} + 1)) \dagger \\
j = j + 1; \\
\quad \{\left(\left(\forall x \left(((1 \leq x) \wedge (x < j)) \rightarrow ((R[x] = r_{n+1-x}) \wedge (R[n+1-x] = r_x))\right)\right. \\
\quad \quad \wedge \left(\left((j \leq x) \wedge (x \leq \frac{n+1}{2})\right) \rightarrow ((R[x] = r_x) \wedge (R[n+1-x] = r_{n+1-x}))\right)\}\} \wedge (j \leq (\frac{n}{2} + 1)) \dagger \\
\quad \text{Assignment} \\
\quad \{\left(\left(\forall x \left(((1 \leq x) \wedge (x < j)) \rightarrow ((R[x] = r_{n+1-x}) \wedge (R[n+1-x] = r_x))\right)\right. \\
\quad \quad \wedge \left(\left((j \leq x) \wedge (x \leq \frac{n+1}{2})\right) \rightarrow ((R[x] = r_x) \wedge (R[n+1-x] = r_{n+1-x}))\right)\}\} \wedge ((j \leq (\frac{n}{2} + 1)) \wedge ((2 \cdot j) > n)) \dagger \\
\quad \text{Partial-While} \\
\quad \{\left(\forall x (((1 \leq x) \wedge (x \leq n)) \rightarrow (R[x] = r_{n+1-x}))\right)\} \wedge ((j \leq (\frac{n}{2} + 1)) \wedge ((2 \cdot j) > n)) \dagger \\
\quad \text{Implied}(b)
\end{array}$$


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