

Natural Deduction Proof Questions (Try to avoid using derived rules!)

1. $\{(\exists x P(x)), (\forall x (P(x) \rightarrow Q(x)))\} \vdash (\exists x Q(x))$
2. $\{(\exists x (P(x) \vee Q(x)))\} \vdash ((\exists x P(x)) \vee (\exists x Q(x)))$
3. $\{((\forall x P(x)) \rightarrow (\exists x R(x))), (\forall x (P(x) \wedge Q(x)))\} \vdash (\exists x R(x))$
4. $\{(\forall x (P(x) \rightarrow Q(x)))\} \vdash ((\forall x P(x)) \rightarrow (\forall x Q(x)))$
5. $\{(\forall x (P(x) \rightarrow Q(x))), (\exists x P(x))\} \vdash (\exists x Q(x))$
6. $\{(\forall x (Q(x) \rightarrow R(x))), (\exists x (P(x) \wedge Q(x)))\} \vdash (\exists x (P(x) \wedge R(x)))$
7. $\{(\forall x (\forall y (P(x) \rightarrow Q(y))))\}, (\exists x P(x))\} \vdash (\forall z Q(z))$

De Morgan's Laws in Predicate Logic:

8. $\{(\neg(\exists x P(x)))\} \vdash (\forall x (\neg P(x)))$
9. $\{(\forall x (\neg P(x)))\} \vdash (\neg(\exists x P(x)))$
10. $\{(\exists x (\neg P(x)))\} \vdash (\neg(\forall x P(x)))$
11. (Assignment problem!) $\{(\neg(\forall x P(x)))\} \vdash (\exists x (\neg P(x)))$

Rules of Natural Deduction

*Connective
or Quantifier*

Introduction Rule(s)

Elimination Rule(s)

\wedge

$$\frac{\alpha \quad \beta}{(\alpha \wedge \beta)} \wedge i$$

$$\frac{(\alpha \wedge \beta)}{\alpha} \wedge e \quad \frac{(\alpha \wedge \beta)}{\beta} \wedge e$$

\vee

$$\frac{\alpha}{(\alpha \vee \beta)} \vee i \quad \frac{\alpha}{(\beta \vee \alpha)} \vee i$$

$$\frac{(\alpha \vee \beta) \quad \boxed{\begin{array}{c} \alpha \\ \vdots \\ \gamma \end{array}} \quad \boxed{\begin{array}{c} \beta \\ \vdots \\ \gamma \end{array}}}{\gamma} \vee e$$

\rightarrow

$$\frac{\boxed{\begin{array}{c} \alpha \\ \vdots \\ \beta \end{array}}}{(\alpha \rightarrow \beta)} \rightarrow i$$

$$\frac{\alpha \quad (\alpha \rightarrow \beta)}{\beta} \rightarrow e$$

\neg

$$\frac{\boxed{\begin{array}{c} \alpha \\ \vdots \\ \perp \end{array}}}{(\neg \alpha)} \neg i$$

$$\frac{\alpha \quad (\neg \alpha)}{\perp} \neg e$$

\perp

(same as $\neg e$)

$$\frac{\perp}{\alpha} \perp e$$

$\neg\neg$

(derived)

$$\frac{(\neg(\neg \alpha))}{\alpha} \neg\neg e$$

\forall

$$\frac{\boxed{\begin{array}{c} u \text{ fresh} \\ \vdots \\ \alpha[u/x] \end{array}}}{(\forall x \alpha)} \forall i$$

$$\frac{(\forall x \alpha)}{\alpha[t/x]} \forall e$$

\exists

$$\frac{\alpha[t/x]}{(\exists x \alpha)} \exists i$$

$$\frac{(\exists x \alpha) \quad \boxed{\begin{array}{c} \alpha[u/x], u \text{ fresh} \\ \vdots \\ \beta \end{array}}}{\beta} \exists e$$

Derived Rules

$$\frac{(\alpha \rightarrow \beta) \quad (\neg \beta)}{(\neg \alpha)} \text{MT}$$

$$\frac{\alpha}{(\neg(\neg \alpha))} \neg\neg i$$

$$\frac{\boxed{\begin{array}{c} (\neg \alpha) \\ \vdots \\ \perp \end{array}}}{\alpha} \text{PBC}$$

$$\frac{}{(\alpha \vee (\neg \alpha))} \text{LEM}$$