

Rules of Natural Deduction

Propositional Logic

<i>Connective or Quantifier</i>	<i>Introduction Rule(s)</i>	<i>Elimination Rule(s)</i>
\wedge	$\frac{\alpha \quad \beta}{(\alpha \wedge \beta)} \wedge i$	$\frac{(\alpha \wedge \beta)}{\alpha} \wedge e \quad \frac{(\alpha \wedge \beta)}{\beta} \wedge e$
\vee	$\frac{\alpha}{(\alpha \vee \beta)} \vee i \quad \frac{\alpha}{(\beta \vee \alpha)} \vee i$	$\frac{(\alpha \vee \beta) \quad \boxed{\begin{array}{c} \alpha \\ \vdots \\ \gamma \end{array}} \quad \boxed{\begin{array}{c} \beta \\ \vdots \\ \gamma \end{array}}}{\gamma} \vee e$
\rightarrow	$\frac{\boxed{\begin{array}{c} \alpha \\ \vdots \\ \beta \end{array}}}{(\alpha \rightarrow \beta)} \rightarrow i$	$\frac{\alpha \quad (\alpha \rightarrow \beta)}{\beta} \rightarrow e$
\neg	$\frac{\boxed{\begin{array}{c} \alpha \\ \vdots \\ \perp \end{array}}}{(\neg \alpha)} \neg i$	(same as $\perp i$)
\perp	$\frac{\alpha \quad (\neg \alpha)}{\perp} \perp i$	$\frac{\perp}{\alpha} \perp e$
$\neg\neg$	(derived)	$\frac{(\neg(\neg \alpha))}{\alpha} \neg\neg e$

Derived Rules

$\frac{(\alpha \rightarrow \beta) \quad (\neg \beta)}{(\neg \alpha)} \text{ MT}$	$\frac{\alpha}{(\neg(\neg \alpha))} \neg\neg i$
$\frac{\boxed{\begin{array}{c} (\neg \alpha) \\ \vdots \\ \perp \end{array}}}{\alpha} \text{ PBC}$	$\frac{}{(\alpha \vee (\neg \alpha))} \text{ LEM}$

Natural Deduction Proof Questions (Try to avoid using derived rules!)

1. $\{(\alpha \rightarrow \beta), (\neg\beta)\} \vdash (\neg\alpha)$
2. $\{\alpha\} \vdash (\neg(\neg\alpha))$
3. $\{((\neg\alpha) \rightarrow \perp)\} \vdash \alpha$
4. $\emptyset \vdash (\alpha \vee (\neg\alpha))$
5. $\{(p \rightarrow q)\} \vdash ((r \vee p) \rightarrow (r \vee q))$
6. $\{((p \wedge q) \rightarrow r)\} \vdash (p \rightarrow (q \rightarrow r))$
7. $\{(p \rightarrow (q \rightarrow r))\} \vdash ((p \wedge q) \rightarrow r)$
8. $\{(p \wedge (q \vee r))\} \vdash ((p \wedge q) \vee (p \wedge r))$
9. $\{(p \rightarrow (q \rightarrow r)), p, (\neg r)\} \vdash (\neg q)$
10. $\{((p \wedge (\neg q)) \rightarrow r), (\neg r), p\} \vdash q$
11. $\{(\neg(p \rightarrow q))\} \vdash (q \rightarrow p)$
12. $\{((\neg\alpha) \wedge (\neg\beta))\} \vdash (\neg(\alpha \vee \beta))$
13. $\{(\neg(\alpha \vee \beta))\} \vdash ((\neg\alpha) \wedge (\neg\beta))$
14. $\{((\neg\alpha) \vee (\neg\beta))\} \vdash (\neg(\alpha \wedge \beta))$
15. $\{(\neg(\alpha \wedge \beta))\} \vdash ((\neg\alpha) \vee (\neg\beta))$