# CS 137 Part 4

## Structures and Page Rank Algorithm

October 6th, 2017

# Structures

- Structures are a compound data type.
- They give us a way to group variables.
- They consist of named member variables and are stored together in memory.

# Example

For example, suppose we wanted to model a clock; we would want the hours and minutes. Let's call this `tod` for time of day.

```
struct tod{
  int hours;
  int minutes;
};
```

We could then use our struct to create instances of the time. For example

- `struct tod now = {16,50};`
- `struct tod later = {.hours = 18};`

# Full Code

```c
#include <stdio.h>
struct tod{
  int hours;
  int minutes;
};
void todPrint(struct tod when) {
  printf("%0.2d:%0.2d\n",
      when.hours, when.minutes);
}
int main(void){
  struct tod now = {16,50};
  struct tod later = {.hours = 18};
  later.minutes = 1;
  todPrint(later);
  return 0;
}
```

# More on Structures

- We can even return structures as well in much the same way as you would expect.
- Example: `struct tod todAddTime(struct tod when, int hours, int minutes)`
- Try to code this example. What issues arise from doing the naive thing?

# Example

```
struct tod todAddTime(struct tod when,
    int hours, int minutes) {
  when.minutes += minutes;
  when.hours += hours + when.minutes / 60;
  when.minutes %= 60;
  when.hours %= 24;
  return when;
}
int main(void) {
  struct tod now = {16, 50};
  now = todAddTime(now, 1, 10);
  todPrint(now);
  return 0;
}
```

# Reminder

- When passing structs to functions, these values are also passed by value.
- If you wanted to modify the original struct in memory, you would need to pass a pointer to it and then modify the contents of the pointers (more on this later).

# Simplification

To make life easier, we can use a typedef to help create structures

```
#include <stdio.h>
typedef struct{
  int hours;
  int minutes;
} tod;
int main(void) {
  tod now = {14,40}; //instead of struct tod
  return 0;
}
```

# Example

Build a Video Game Character struct. Call this `vgchar`. It should have an identification number, x and y positions starting at the origin, a power level and a defense level. Then, create some of the following functions:

- Move up, down, left, right
- Fight (between two characters, take the power levels subtract the opponents defense levels and the higher wins)
- Change ID which takes in a new integer ID number.

# Page Rank

- How does Google's Searching Algorithm Work?
- Main idea: It crawls the web, indexes words on each page and then uses the index to find matches and sort by how relevant it is.
- Algorithm is due to Sergey Brin and Larry Page in their paper "The Anatomy of a Large-Scale Hypertextual Web Search Engine"
- See `http://infolab.stanford.edu/~backrub/google.html`
- Major idea: Pages with lots of links to them should be classified as good (otherwise why do they have lots of links?)

# Definition of the Rank of a Page Version 1

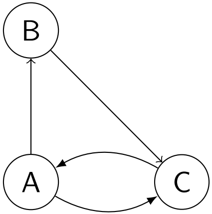
- For a page $P$, let $Q_i$ be all of the pages that link to $P$ for $1 \leq i \leq M$ with $M$ a nonnegative integer.
- Then, the rank of a page $P$, denoted by $r(P)$ is

$$r(P) = \sum_{i=1}^{M} \frac{r(Q_i)}{|Q_i|}$$

  where $|Q_i|$ is the number of outbound links on page $Q_i$.
- We will also normalize throughout so that their sum is one.

# Simple Example

Suppose the internet consisted of three pages, $A, B$ and $C$. Below, the arrows indicate which pages go to which other pages.




Using the formula on the previous page, we have

$$r(A) = \frac{r(C)}{1} \qquad r(B) = \frac{r(A)}{2} \qquad r(C) = \frac{r(A)}{2} + \frac{r(B)}{1}$$

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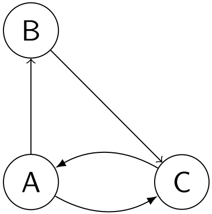


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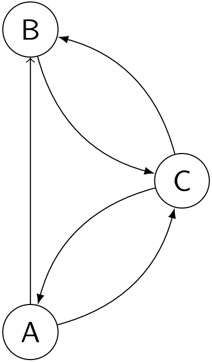
$$r(A) = \frac{r(C)}{1} \qquad r(B) = \frac{r(A)}{2} \qquad r(C) = \frac{r(A)}{2} + \frac{r(B)}{1}$$

Solving gives $r(A) = 2r(B) = r(C)$. Normalizing so that $r(A) + r(B) + r(C) = 1$ gives $r(A) = 0.4$, $r(B) = 0.2$, $r(C) = 0.4$

# Clicker

Given the same normalization as before, what is the value of $r(A)$ for the following?




a) 2/9
b) 1/3
c) 4/9
d) 5/9
e) None of the above

# Random Surfer Model

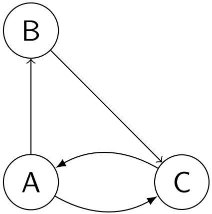
- What Google does is slightly more sophisticated than this.
- Suppose we had a web surfer that:
  - with probability $\delta$ goes to a link from the current page
  - with probability $1 - \delta$ goes to a random page
- Define the randomized page rank of a page $r_\delta(P)$ to be

$$r_\delta(P) = \frac{1-\delta}{N} + \delta \sum_{i=1}^{M} \frac{r(Q_i)}{|Q_i|}$$

where $N$ is the number of pages on the web.

# Simple Example

With this model, let's revisit our example with $\delta = 0.8$.




Using the formula on the previous page, we have

$$r(A) = \frac{1}{15} + \frac{4r(C)}{5} \qquad r(B) = \frac{1}{15} + \frac{2r(A)}{5}$$

$$r(C) = \frac{1}{15} + \frac{2r(A)}{5} + \frac{4r(B)}{5}$$

# Solving

- With a little patience and the usual normalization, we can solve this system to see that

$$r(A) = \frac{61}{159} \approx 0.38365\ldots \qquad r(B) = \frac{35}{159} \approx 0.22013$$
$$r(C) = \frac{21}{53} \approx 0.39623$$

  via say Gaussian Elimination or even Cramer's Rule.
- However, if $N$ is large... say $N > 10^9$ this is very infeasible.
- A trick that is often used is to use a fixed point iteration process called Jacobi's Method!

# Jacobi's Method

- Begin with a solution $(r_0(A), r_0(B), r_0(C)) = (1/3, 1/3, 1/3)$.
- Compute

$$r_1(A) = \frac{1}{15} + \frac{4r_0(C)}{5}$$
$$r_1(B) = \frac{1}{15} + \frac{2r_0(A)}{5}$$
$$r_1(C) = \frac{1}{15} + \frac{2r_0(A)}{5} + \frac{4r_0(B)}{5}$$

Repeat with $(r_1(A), r_1(B), r_1(C))$ until you're happy!

# For Our Example

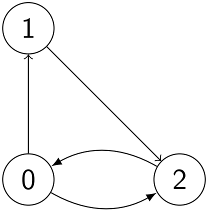
| Iteration | r(A) | r(B) | r(C) |
|---|---|---|---|
| 0 | 0.33333 | 0.33333 | 0.33333 |
| 1 | 0.33333 | 0.20000 | 0.46667 |
| 2 | 0.44000 | 0.20000 | 0.36000 |
| 3 | 0.35467 | 0.24267 | 0.40267 |
| $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ |
| 19 | 0.38364 | 0.22013 | 0.39623 |

This is a good approximation after 20 iterations. Note that for the web this would take thousands of iterations (but is still faster than solving exactly!)

# Representation in Memory

We will use structs to help encode this

```
#include <stdio.h>
typedef struct {
  int src, dst;
} link;
int main(void) {
  link l[] = {{0,1}, {0,2}, {1,2}, {2,0}};
  return 0;
}
```



# Main Ideas Of Next Slide

- Compute out links
- Make initial guess and store in `r`, normalized
- Perform the iterative process in `s` and update in `r`.

# PageRank.c

```
void pagerank (link l[], int n_link, double r[],
  int n_page, double delta, int n_iter){
  double s[n_page];
  int out[n_page];
  for (int i = 0; i < n_page; i++) out[i] = 0;
  for (int j = 0; j < n_link; j++)
    out[l[j].src]++;
  for (int i = 0; i < n_page; i++)
    r[i] = 1.0 / n_page;
  for (int k = 0; k < n_iter; k++) {
      for (int i = 0; i < n_page; i++)
        s[i] = (1.0 - delta) / n_page;
      for (int j = 0; j < n_link; j++)
        s[l[j].dst] +=
          (r[l[j].src]/out[l[j].src])*delta;
      for (int i = 0; i < n_page; i++)
        r[i] = s[i]; }}
```

# PageRank.c (Continued)

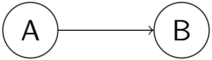
```
#include <stdio.h>
//Insert page rank code from before here
void main () {
  link l[] = {{0,1},{0,2},{1,0},{2,1}};
  double r[3];
  pagerank(l, sizeof(l)/sizeof(l[0]), r,
    sizeof(r)/sizeof(r[0]), 0.80, 20);
  for (int i = 0; i < 3; i++)
    printf("%g\n", r[i]);
}
```

## Final Warning - Sinks

- What happens if a page has no outgoing links?
- We call such pages sinks. With a sink, the only way to escape it is to leave randomly.
- With sink nodes, probabilities become lost.
- Let's see this explicitly with an example.

# An Example

Consider the following with $\delta = 0.8$:




- The page rank equations for this are

$$\begin{aligned} r(A) &= \frac{1-\delta}{2} = 0.1 \\ r(B) &= \frac{1-\delta}{2} + \delta \cdot \frac{r(A)}{1} = 0.18 \end{aligned}$$

- Notice that these values do not add up to 1 and will never change after iterations.

# Fixes

We can fix this problem in a few ways:

- Renormalize the final answer (this is bad because it over-estimates the page rank for nodes with many in-links)
- Connect each sink node to all other nodes (this is expensive)
- Connect each sink node without actually connecting them:

$$r_\delta(P) = \frac{1-\delta}{N} + \delta \sum_{i=1}^{M} \frac{r(Q_i)}{|Q_i|} + \delta \sum_{i=1}^{M_0} \frac{r(S_i)}{N}$$

where the first sum contains no sinks and the second sum is over all sinks ($M_0$) is the total number of sinks.

# Other Problems

- What about cyclic links?
- Hoarding - websites linking to each other to boost page rank.
- We won't discuss these (or other issues) in this course.