

## CO 480 Lecture 17

The French Revolution, Sophie Germain and History of Women  
in Mathematics

July 4th, 2017

## This Day in History July 4th, 1843

Liouville began an address to the Academy of Sciences with the words: "I hope to interest the Academy in announcing [that in] the papers of Evariste Galois I have found a solution, as precise as it is profound, of this beautiful problem: whether or not [the general equation of fifth degree] is solvable by radicals." This work of Galois was published in 1846.

# Announcements

- Editorial Review is due Thursday at 2:30pm.
- Assignment 4 is due Thursday July 20th at 2:30pm.
- Final Edition will be due on Tuesday July 25th
- Final Quiz will be on Tuesday July 25th.

# Today's Lecture

- In the next three lectures, we will be talking about Sophie Germain and Adrien-Marie Legendre (later Gauss who will also be important)
- Both lived during the French Revolution
- I will start this topic with an extremely limited and brief overview of the French Revolution. Our library has a plethora of resources on the topic.
- The Revolution has many debates surrounding it and I will do my best to give an objective view of it (and some parallels to modern day)



## France from 1780-1789

- France consisted of three estates and the monarchy.
- This was called the *Old Regime* or *Ancien Régime*.
- Estates were the clergy, the nobles and the others (wage labourers, serfs/peasants, bourgeoisie (middle class) etc.)
- Primarily, citizens lived in rural areas which was 10 times what it was today [McP02, p. 4].

# Three Estates



## The First Estate - The Clergy

- Paid little in taxes [Jon10, p. 5]
- Controlled education, employed many, carried political weight [Dav09, p. 3]
- Consisted of less than 1% of the population [McP02, p. 13]
- Gained its wealth from tithings (literally translated to a tenth of what people make) - approximately *150 million livres* a year. Paid 3% voluntarily to the monarchy
- Owned 10% of all the land (which generated *130 million livres* a year in revenue)
- Catholic Church basically was a monopoly.
- Bishop of Strasbourg annual stipend of 450,000 *livres* (priests made 750) [McP02, p. 13]

## The Second Estate- The Nobles

- Like the clergy, paid little in taxes [Jon10, p. 5]
- Slightly larger than the Clergy.
- Often fought in wars.
- Participated in elite position in the Catholic Church [McP02, p. 16]
- Owned a third of the land but worse still, owned seigneurial rights on the rest (usually between  $1/12$  and  $1/6$  (even up to  $1/4$ ) of the harvest)
- Gave nothing to the monarchy
- Note: While people were sometimes exempt from taxes, territories as well were often exempt (eg. Brittany, Poitou, etc. [Jon10, p. 5])

## The Third Estate - The Others

- Estimates ranges anywhere from 97% – 99% of the population [McP02, p. 18]
- Only people relied on for taxes for the monarchy known as the *taille*.
- Top consisted of educated, professionals and merchants [Dav09, p. 4]
- Labourers, worked the land and other jobs as needed.
- Middle class (bourgeoisie) owned about 25% of land.

# The Monarchy

- Stood at the top; seen as France's natural leader [Dav09, p. 6].
- Were seen as those in direct contact with God and doing God's work.
- Constitution by which the king governed France was not written down, rather it was customary [McP02, p. 19].
- Ruled from the Palace of Versailles (more later).
- Put the country into massive debt by fighting wars (more on this later).

# House of Bourbon

- Noble family - ruled France (1589-1792)
- Henri VI (1589-1610), Louis XIII (1610-1643), Louis XIV (Sun King - 1643-1715), Louis XV (1715-1774), Louis XVI (1774-1792)
- Louis XVI [fat ill-bred boy] married Austrian arch-duchess - Marie Antoinette - when he was 15 years old (she 14). [Allies now: Austria and France]
- Ruled in a style rooted in negotiation (even though they were under no obligation to do so) [Jon10, p. 3]

**Louis XVI**



[https://en.wikipedia.org/wiki/File:Duplessis\\_-\\_Louis\\_XVI\\_of\\_France,\\_oval,\\_Versailles.jpg](https://en.wikipedia.org/wiki/File:Duplessis_-_Louis_XVI_of_France,_oval,_Versailles.jpg)

# Palace of Versailles



(Wikimedia Commons)



# Versailles

- Located 20km southwest of Paris.
- Moved the capital from Paris to Versailles.
- Originally built by Louis XIII in 1623 expanded by Louis XIV (1661-1678 expansion)
- Originally planned to be an occasional residence but Louis XIV moved the capital here.
- Insanely expensive structure - funded by taxes from citizens and New France (Canada!)
- Current valuation: \$50.7 billion USD<sup>1</sup>.

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<sup>1</sup>

<https://www.msn.com/en-in/money/photos/what-the-worlds-most-valuable-palaces-are-really-worth/ss-AAjH5E8#image=26>

# Wars

- Seven Years War (1756-1763) - Mainly France vs Britain but eventually France and Austria versus Britain and Prussia (Russia played both sides) - Spain and Saxony [landlocked free state in Germany] also entered. (Recall: Euler)
- Issues over territory in North America.
- Treaty of Paris between France, Spain and Great Britain and the Treaty of Hubertusburg between Saxony, Austria and Prussia, in 1763
- Fighting the american revolution (1778 - revenge against Britain) costs 2 billion *livres* and by 1788 the total debt was 5 billion *livres* [Jon10, p. 10].
- Debt eventually became crippling (cannot just continue to increase taxes forever).

## Dealing With Debt

- Jacques Necker, Genevan banker, finance minister from 1777-1781 and Principal Minister in 1788. [McP02, p. 18]
- People's voice in Versailles.
- Believed that government's job was to ensure people had enough to eat and drink.
- *Compte rendu au roi* (1781) document that finally made transparent government expenditures [Dav09, p. 15-17]
- Unpopularity with Marie Antoinette coupled with above likely cause of his termination in 1781 [Dav09, p. 16]



(Wikimedia Commons)

# Growth

- Population in France grew from about 24.5 million in 1750 to 28 million in 1780, largely due to good harvests [McP02, p. 13].
- However, people's food supplies were always under threat of a poor harvest year or, more worrisomely, from taxes, seigneurial dues (landlord dues) and tithings which took away anywhere from a quarter to a third of the total yield [McP02, p. 13].

# Causes of the War

- Lots of speculation.
- Two major causes (in my opinion)
  1. Hyperinflation (French consumed 2 pounds of bread a day [Lynn Hunt - UCLA] - poor harvest years caused cost of flour to double - soon equal to a month's earnings for one loaf)
  2. Poor political structure (Each estate basically had a 1/3rd say meaning 99% of the population could be easily outvoted by 1%)
- Some argue that the "Age of Enlightenment" caused new ideas to flow amongst the populous. (Thinking for yourself vs. being told what to think)
- December 1788 - Necker recommends that the voting power of the third estate be doubled to give the people a say

# Estates General

- General assembly of the three estates
- Agreed to meet On May 5th, 1789 (previous time: 1614)
- Voting was done by estate still (the point of the meeting was to change this to by head).

## Timeline of Events [McP02]

- May 28th - Third Estates starts meeting on their own.
- June 10th - Verified its [the Third Estate's] own powers.
- June 13th - Some priests elect to join the Third Estate.
- June 20th - Tennis Court Oath (actually took place on a handball court).
- July 1st - King calls on reinforcements to deal with crowds.
- July 9th - Formally formed National Constituent Assembly.
- July 11th - Sacked Necker (Paris found out on July 12th).
- July 14th - Stormed the Bastille (Paris prison) began demolition.
- August 27th - Declaration of Rights of Man.
- October 5th - October Days. (The March on Versailles)

## Tennis Court Oath

- Third Estate comes to the chambers to find out they've been locked out
- Went next door to a handball court - agree to not stop meeting until they have a new constitution
- Formed the National Assembly [of the people] - eventually the other two Estates become forced to join them.



# Bastille

- Worries that King is plotting against them.
- Assembly wants to take arms but they have no gun powder
- July 14th, sack the Bastille in Paris (prison; torture chamber).
- Form a flag - blue and red for the house of the people separated by white, the colour of the House of Bourbon

# Maximillian Robespierre

- Called the “incorruptible” for his beliefs in his ideals.
- Called out the King for eating in a day what 1000 men should be eating.
- Wanted all estates to be taxed.
- Fought for Freedom of Speech.
- Originally against death penalty.



(Wikimedia Commons)

# Declaration of Rights of Man

- August 27th, 1789
- Revoked censorship
- Made all men equal
- Retained property rights
- Had a strange graded citizenship system whereby people who made more taxes were eligible to vote.
- King is nowhere mentioned
- Robespierre had a hand in this document's creation.



[https://commons.wikimedia.org/wiki/File:Declaration\\_of\\_the\\_Rights\\_of\\_Man\\_and\\_of\\_the\\_Citizen\\_in\\_1789.jpg](https://commons.wikimedia.org/wiki/File:Declaration_of_the_Rights_of_Man_and_of_the_Citizen_in_1789.jpg)

## October Days [McP02, p. 60-61]

- Alias: The March on Versailles
- 1789 harvest was poor; women marched on Versailles
- Had blessings from Robespierre
- Marquis de Lafayette, head of National Guard in Paris led the women from Paris (feared mutiny and death if he did not join)
- His goal was to be a mediator (protect king attempt to keep some order)

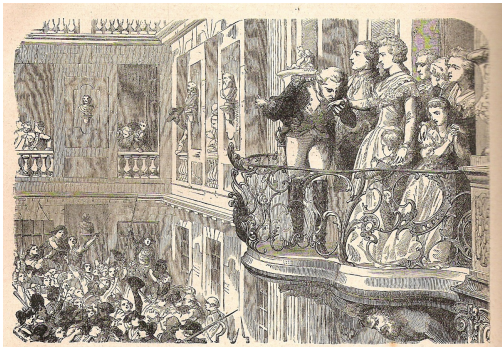


## October Days [Wikb]

- Arrived to Versailles on October 5th (6 hour march).
- King Louis XVI first entreated a small group of women promising them to take food from their stores.
- Some were satisfied and left for Paris however most still wanted more.
- At 6:00am October 6th, a group of women found a small gate that was ungarded and stormed to the chambers ransacking the bed.
- Lafayette urged the king to address the angry mob which was now 60000 strong.
- The mob shouted “Vive le roi” to the relief of the two men and Louis XVI in compromise with the mob.

## Return of the King [Wikb]

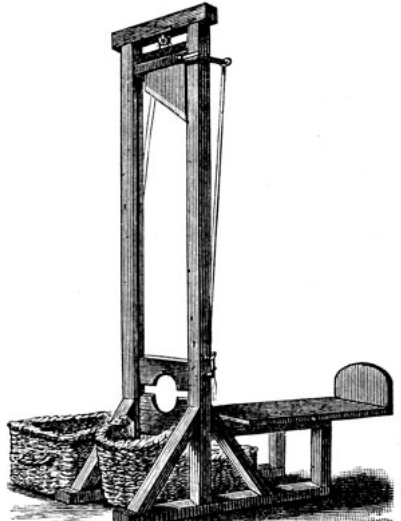
- The mob negotiated that the king and family return to Paris (back to Tuileries Palace)
- Brought all the flour and store house grains from Versailles
- Many soldiers died and some had their heads placed at the end of a pike



(Public Domain Picture)

# Guillotine

- Decapitation was reserved for nobility.
- Named after Dr. Joseph-Ignace Guillotin - believed in capital punishment by decapitation .
- Proposed to the national assembly on October 10th, 1789.
- Prototype conceived by Antoine Louis, together with German engineer Tobias Schmidt.
- The equalizer - wanted everyone to have the same death.



## Louis Escapes [Jon10, p. 47-48]

- Under the cover of night on June 21st, 1791, King Louis XVI and family try to escape Paris to Austria to seek help in controlling the riots.
- Spotted in Varennes, miles from the Austrian border, by local town official.
- Louis XVI was hoping for sympathy from the locals however many supported the revolution and they were returned to the palace in Paris
- Psychologically damaging that monarch was seen abandoning its people.



## Jacobins and Girondins

- Influential political club - mainly deputies from Brittany at first (added Parisians later).
- Girondins (moderate) were a subset of the Jacobins (more radical). Girondins dominated at first. [Dav09, p. 56-59]
- Decided on the national agenda. Responsible for foreign affairs.
- In 1792, Robespierre became more of a leader.
- October 1793, 21 Girondins were executed helped to pass new constitution.

## War Against Austria - 1792

- April 20th, 1792 - France declares war against Austria [Dav09, p. 65]
- Fearing that Austria might come to monarch's aide, France (the National Assembly) declares war on Austria. (Prussia allies with Austria)
- Robespierre was against it - felt France would lose.
- Very awkward for monarch.

# Brunswick Manifesto

- Duke of Brunswick - leader of Prussian and Austrian armies
- July 25th, 1792
- Claimed if any harm were to come to the monarch, then harm would come to its citizens.
- Spurred rage in the revolution - August 10th, 1792 - 27,000 citizens sacked Tuileries Palace and took the king.
- King takes refuge with National Assembly - monarchy is no more. Republic is born.
- Danton urges people to return to join the war.

# Marat

- Born Jean-Paul Marat May 24th, 1743.
- Newspaper - *L'ami du Peuple* - The People's Friend.
- Fan of bloodshed - Coins the name "guillotine".
- Often ordered the death of people using arguments along the lines of "There will be peace if we kill just one more person".



Wikimedia Commons

## More on Marat

- 1792, encourage France to kill the imprisoned who were against the revolution (they are losing the war).
- September massacre (short impromptu trials quickly ended in death)
- Revolution needs leader - Robespierre fills the void. Puts king on trial. (Treason)
- Girondin - wanted Louis XVI spared. Jacobins outnumbered them and wanted blood.
- January 23rd, 1793 - Louis XVI killed by guillotine (Robespierre)

## Marat Goes Crazy

- Girondins call for less bloodshed but Marat wants more
- Begins naming people who should die for suspected plots against the revolution.
- Charlotte Corday took matters into her own hand and goes to kill Marat on July 13th, 1793 (Marat was in a medicinal bath)
- Wanted to kill Marat so that peace would return to France.
- Marat becomes a martyr in death - more violence ensues.
- Later Marie Antoinette also executed - October 16, 1793.

# The Death of Marat Jacques-Louis David (1793)



# Documentary

<https://www.youtube.com/watch?v=5pXxoyk5w0o>

**The Terror and Dechristianization: 1:11:10 - 1:19:10**



## This day in History

**7-6-1785** Decimal system of money with the dollar as unit adopted by the Continental Congress of the United States of America.

**7-6-1854** Georg Simon Ohm died in Munich, Germany. Ohm's Law describing conduction of electricity in a circuit was part of a complete theory of electricity deemed too mathematical by contemporary physicists.

## Terror (Robespierre's definitions)

- A good citizen no longer just pays taxes - must participate in war.
- February 5th, 1794 - Robespierre gives a speech outlining philosophy:

*Terror without virtue is disastrous.*

*Virtue without terror is powerless.*

- Dantonese refute Robespierre but he orders them killed (Danton's only regret is that he will die before Robespierre)
- June 27th, 1794 - Robespierre's last speech of threats. Announces new names but fails to reveal list in the speech. Everyone is scared so they capture him.
- Robespierre found jaw shattered - believed to be attempted suicide.
- Sent to guillotine. July 28th, 1794. Ends terror.

# Final Story - Joseph Fourier

- Of the “Fourier Series”  
(representing functions by trigonometric series)
- His outward criticism of corruption in 1794 led him to be arrested and sentenced to the guillotine [Mus05, p. 261].
- Pled his case to Robespierre but to no avail.
- However, Robespierre was executed on July 28th, 1794 and Fourier was immediately released.



<https://en.wikipedia.org/wiki/File:Fourier2.jpg>

# Academies

- We've mentioned three major academies, The one in St. Petersburg, Berlin and briefly the one in Paris.
- I have spoken little about the Paris Academy (formally, the French Academy of Sciences) which I will do now before speaking about Sophie Germain

# French Academy of Sciences



Source: <http://www.princeton.edu/~his291/Jpegs/Academie.JPG> (Public Domain)

# French Academy of Sciences

- *Académie royale des sciences de Paris*
- During the rule of King Louis XIV, chief minister Jean-Baptiste Colbert creates the Paris Academy in 1666. [Eul]
- Royal society of London was founded in 1660 and France was possibly concerned with geopolitical issues.
- Meetings occurred twice a week; Wednesdays mathematics was discussed and Saturdays physical sciences were discussed in the King's Library. [Eul]
- Academy had no statues during its first 30 years.
- Academy was to stay away from political and religious issues (focus mainly on arts and sciences).

## Issues [Eul]

- Intellectual property rights were a big issue (again this is something worth fighting for!)
- Crown claimed that all proceedings from the Academy were its own private property.
- Constituents would break these rules and publish on the side (hence the academy was meeting but not producing)
- In 1699, King Louis XIV gave the Academy its first rules and constituents began publishing in their in house journals *Mémoires* (this was how pensions were determined).
- 1720 started its annual (shortly after reduced to biannual) prize competition (recall Euler).
- Closed 1793-1795 due to the Revolution.

## A Contentious Point

- Many sources claim that women were not allowed into the academy for almost 3 decades [Wika], [Wha03, p. 105].
- Whaley even claims “Interestingly, the site provides a list of important dates in the Academy’s history, but the date of admittance of women as members is not there”
- This is contentious because there were no formal statutes that actually banned women from the academy [Pet99, p. 383]
- However, what is true is that women were naturally barred from the Academy, given that they often received little to no formal educational training and could therefore seldom compete with men at their time; not to mention formal unwritten traditions often kept them from joining these establishments.



# Women in Mathematics During 1700s France

- Briefly mentioned Hypatia (370 AD) and Agnesi (1718-1799)
- Two major players: Émilie Du Châtelet and Sophie Germain
- Will focus on Sophie Germain but will compare and contrast these two women's paths to mathematics
- Émilie (on the right) was born in 1706 and died in 1749.



[https://commons.wikimedia.org/wiki/File:Emilie\\_Chatelet\\_portrait\\_by\\_Latour.jpg](https://commons.wikimedia.org/wiki/File:Emilie_Chatelet_portrait_by_Latour.jpg)

## Life of Du Châtelet [Pet99, p. 383]

- Born Gabrielle-Émilie de Brteuil into a wealthy aristocracy (service to King Louis XIV); received excellent education.
- Know little of her childhood.
- Tutored by father and they had a large library.
- Hosted a *salon* (more on this later) which Voltaire attended.
- Studied Latin, Italian, English, mathematics and sciences
- Had a prolonged affair with Voltaire [Mac90, p. 347].
- Check out <https://www.youtube.com/watch?v=6KkGU5R9QVY>

## Work of Du Châtelet

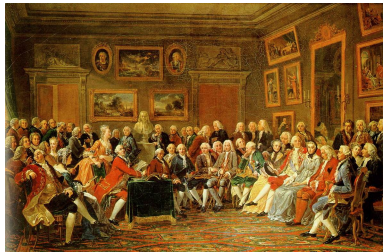
- Spent most of her later years translating Newton's *Principia* into French (only known French version).
- Finished after death by her friend and mentor Alexis Claude Clairaut.
- In 1738, wrote a paper refuting Voltaire's views on the nature and propagation of fire; prize however went to Euler's *On the Nature and the Properties of fire*.
- Paper was published however. (Submitted anonymously but not really since was on letterhead, handwriting etc.)
- Rumours that she became pregnant by a lover not her husband or Voltaire at the age of 42 and died shortly thereafter.  
[Mac90, p. 348]



<http://images.clipartpanda.com/flame-clipart-pc5AGnRcB.svg>

## Salons in France [Wha03, p. 82-84,106-108]

- The education of good manners, ideas and social norms
- Women attending were at first noble later middle classed (after the Fronde in 1648-1653, a series of rebellions against King Louis XIV for rising taxes)
- Issues for women included education and their rights to choose their own husband.
- Debate over women's role - Just support? To disseminate their own new ideas? To meet men?



[https://en.wikipedia.org/wiki/File:Salon\\_de\\_Madame\\_Geoffrin.jpg](https://en.wikipedia.org/wiki/File:Salon_de_Madame_Geoffrin.jpg)

## Main Event - Sophie Germain

- “It matters little who first arrives at an idea, rather what is significant is how far that idea can go” (Germain)

- See

<http://www.azquotes.com/quote/1371204>  
for more.

- “Sophie Germain proved to the world that even a woman can accomplish something in the most rigorous and abstract of sciences and for that reason would well have deserved an honorary degree.” (Gauss)



(Wikimedia Commons)

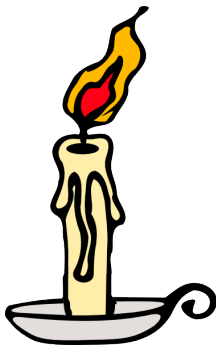
- <http://www.goodreads.com/quotes/1057024-author-sophie-germain-6484914-proved-to-the-world-that-even-a-woman>

## Life of Sophie Germain [Mus05, p. 249-265]

- Born in Paris on April 1st, 1776 (seldom if ever left France).
- Daughter of Ambroise-François and Marie-Madeleine Germain.
- Father was a wealthy silk merchant [Stu79]
- Began her interest in mathematics around 1789 (the heart of the French Revolution) when she was 13.
- Read a story about the death of Archimedes in Montucla's *Histoire des mathématiques* inspiring her to study mathematics [OR][LP10] (Libri).

# Candlelight Mathematics [OR][LP10]

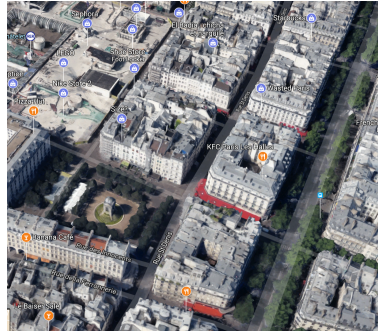
- Libri writes that Germain would wake up in the middle of the night to do mathematics
- Parents removed her fire, clothes and candles from her room
- Undeterred, she awoke under dim lamp light to do mathematics (even with a frozen ink well!)



[http://www.clker.com/cliparts/5/6/c/f/11970930102048171349LostInBrittany\\_Candle.svg](http://www.clker.com/cliparts/5/6/c/f/11970930102048171349LostInBrittany_Candle.svg).  
hi.png

# Sophie and the Revolution

- Her residence, rue St. Denis No. 336 [Stu79] was at the heart of many revolution landmarks [Mus05].
- Revolution forced her to stay home - spent much time in her father's library.



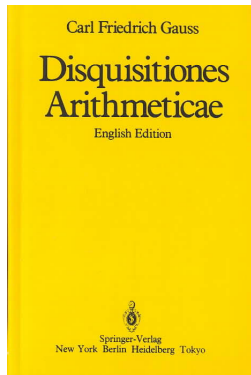


## École polytechnique [Mus05, p. 251-253]

- In 1794, École Centrale des Travaux Publics was formed. Renamed in 1975 as École Polytechnique
- Being a woman, she was not allowed to apply, let alone attend the school.
- Often obtained lecture notes (how, to this day, we do not know)
- Lagrange (1736-1813) was the founding professor of analysis.
- He taught there from May 24th, 1795 until 1799.
- Rumoured to have corresponded with Lagrange but evidence is scarce.
- Also befriended Legendre, *Examineur* from 1799-1816

# Letters to Gauss [Mac90]

- 1801 - Gauss' *Disquisitiones Arithmeticae* is published.
- Sent lots of letters to Gauss under her pseudonym
- In a letter dated February 20th, 1807, Germain revealed to Gauss that she is a woman.



[http://t2.gstatic.com/images?q=tbn:ANd9GcTCmtcygTVkebnEBMI2pr\\_r72maT4VXC\\_E8\\_FF4AsMIhAuvq14n](http://t2.gstatic.com/images?q=tbn:ANd9GcTCmtcygTVkebnEBMI2pr_r72maT4VXC_E8_FF4AsMIhAuvq14n)

## Gauss on Sophie Germain [Mac90]

Letter to Sophie Germain dated April 30th, 1807

*The enchanting charms of this sublime science reveal themselves in all their beauty only to those who have the courage to go deeply into it. But when a person of that sex, that, because of our mores and our prejudices, has to encounter infinitely more obstacles and difficulties than men in familiarizing herself with these thorny research problems, nevertheless succeeds in surmounting these obstacles and penetrating their most obscure parts, she must without doubt have the noblest courage, quite extraordinary talents and superior genius.*

[https://en.wikiquote.org/wiki/Carl\\_Friedrich\\_Gauss](https://en.wikiquote.org/wiki/Carl_Friedrich_Gauss)

## Reply from Gauss April 20th, 1807 [Mac90]

- Gauss also refers to a question Sophie Germain asked. She conjectured the following

### Germain's Conjecture

If  $a^n + b^n$  can be written in the form  $s^2 + nt^2$  (for  $a, b, s, t, n$  integers), then  $a + b$  can also be written as  $u^2 + nw^2$ .

- In Gauss' reply he states that this "is put a little too strongly. Here is an example of where it fails:"

$$15^{11} + 8^{11} = 8,658,345,793,967 = (1,595,826)^2 + 11(745,391)^2$$

however  $15 + 8 = 23$  cannot be reduced to the form  $x^2 + 11y^2$ .

- MacKinnon asks:

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however  $15 + 8 = 23$  cannot be reduced to the form  $x^2 + 11y^2$ .

- MacKinnon asks: "Was Gauss a Feminist?"

## Answer?

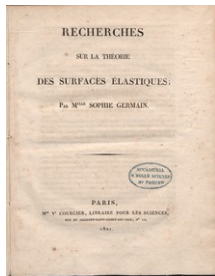
- Probably not. Firstly, Gauss in 7 weeks (received the letter March 12th) managed to come up with this gargantuan example by hand meaning that he probably spent a lot of time on this paper.
- Gauss also wasn't as prolific in French as other languages [Mus05, p. 255], [Stu79] so corresponding with Germain probably took longer.
- MacKinnon points out there are many boring small examples:  $4^2 + 1^2 = 3^2 + 2 \cdot 2^2$  and  $4^4 + 2^4 = 16^2 + 4 \cdot 2^2$  but Gauss likely assumed she wanted  $n > 2$  and  $\gcd(a, b) = 1$  and Gauss chose not to use these.

## In Fact...

- How did Gauss come up with this example?
- Waterhouse [Wat94] gives an explanation as to how Gauss might have discovered this using only material from Gauss' book.
- Boils down to the following [Wat94]:
  1. Take a prime represented by  $3X^2 + 2XY + 4Y^2$  and write it in all ways as a sum  $a + b$ . (First such are 3, 5, 23)
  2. Take another prime  $p$  of the form  $11k + 1$  represented by  $3X^2 + 2XY + 4Y^2$ .
  3. Test whether  $a^{11}$  is congruent to  $-b^{11}$  modulo  $p$ .
- Aside: Sophie's conjecture is true when  $n = 3$  [Mac90].

## Paris Academy of Sciences [Mus05, p. 256-257]

- On the first Monday of 1809, the Academy announced a prize for anyone who could offer an explanation to the vibrating plate experiments as done by German physicist Ernst F. F. Chladni [Pet99, p. 384].
- Judges were Legendre, Laplace, Lagrange, Lacroix and Malus; prize was a gold medal worth 3000 francs.
- In 1809, Legendre became friends with Germain, providing her with access to information and current research (coinciding with her time working on the following problem) [Pet99, p. 386].



[https://commons.wikimedia.org/wiki/File:Germain\\_-\\_R%C3%A9cherches\\_sur\\_la\\_th%C3%A9orie\\_des\\_surfaces\\_%C3%A9lastiques,\\_1821\\_-\\_723413.tif](https://commons.wikimedia.org/wiki/File:Germain_-_R%C3%A9cherches_sur_la_th%C3%A9orie_des_surfaces_%C3%A9lastiques,_1821_-_723413.tif) (Not the original paper - published in 1821)



## Germain's Work

- Germain took on the challenge and began extending the analysis of Euler.
- In January 1811, we know of communications with Legendre on her work [Mus05, p. 258]
- Sophie's entry was the only entry in the competition submitted on September 21st, 1811.

## Germain's Work

- Germain took on the challenge and began extending the analysis of Euler.
- In January 1811, we know of communications with Legendre on her work [Mus05, p. 258]
- Sophie's entry was the only entry in the competition submitted on September 21st, 1811.
- Despite this, Germain's submission was not accepted as a winning submission.
- It was judged inadequate because "the true equations of the movement were not established" but "the experiments presented ingenious results" [Pet99, p. 384].

## Sans Winner...

- With no winner, the competition was extended by 2 more years.
- Lagrange amended Germain's computations to better (though not fully) describe Chladni's experiments.
- Germain for her second entry worked seemingly completely in isolation.
- One hurdle facing Germain was her lack of formal mathematical training [Pet99, p. 384].
- Her second entry in 1813 was also the only entry but was also denied (though she received an honorable mention)
- The contest was reopened for a third time and here on January 8th, 1815 [by now under her own name] the community accepted her paper for the prize (which she declined!)

## Life after 1816 [Pet99, p. 386]

- Even as the first woman to win a price from the Paris Academy, she still wasn't given regular admittance into lectures
- Leftover tickets were first given to academic's wives then later to the public.
- After befriending Fourier (Recall who was alive only by happenstance!) who was then the secretary of the Academy, was she able to regularly obtain tickets

## More from the Academy

- In 1818, the Academy also offered a prize for a proof of Fermat's Last Theorem. This was withdrawn in 1820 [Cen05]
- Sophie Germain, in letters to Gauss and Legendre, spent a lot of effort outlining a plan to prove Fermat's Last Theorem (FLT).
- This plan is often understated in many resources and only recently (2010) did Laubenbacher and Pengelley write a paper outlining in detail Sophie's ingenious ideas with attacking FLT.
- We follow Laubenbacher and Pengelley [LP10] in what follows.

# Major Theorem

## Sophie Germain's Theorem

For an odd exponent  $p$ , if there exists an auxiliary prime  $\Theta$ , that is, a prime such that there are no two nonzero consecutive  $p$ th powers modulo  $\Theta$ , nor is  $p$  itself a  $p$ th power modulo  $\Theta$ , then in any solution to the Fermat equation  $z^p = x^p + y^p$ , one of  $x$ ,  $y$  or  $z$  must be divisible by  $p^2$ .

- This is sometimes called case 1 of Fermat's Last Theorem.
- Sophie Germain only proved that  $p$  must divide one of  $x$ ,  $y$  or  $z$  and Legendre extended it to the aforementioned theorem.
- In a memoir of Legendre in 1823, he published Germain's results and also extended her work to complete the case of FLT when  $p = 5$ , the third known FLT result.[LP10].

## Example

$$p = 3 \text{ and } \Theta = 13$$

Residue	1	2	3	4	5	6	7	8	9	10	11	12
Cubic Residue	1	8	1	12	8	8	5	5	1	12	5	12

Notice that 3 is not in the list and no two of 1, 5, 8, 12 are consecutive. Thus  $p = 3$  and  $\Theta = 13$  satisfies the condition of the previous theorem and hence one of  $x$ ,  $y$  or  $z$  is divisible by  $p^2$ .

# Sophie Germain Primes

## Sophie Germain Prime

A Sophie Germain Prime is a prime  $p$  such that  $2p + 1$  is also a prime.

- First few examples: 2, 3, 5, 11, 23, 29, 41, 53, 83, 89, ...  
<https://oeis.org/A005384>
- Originally looked at auxiliary primes  $\Theta = 2Np + 1$  (easiest case is when  $N = 1$ )
- These [odd] primes trivially satisfy the previous theorem (the only  $p$ th powers are  $\pm 1$  - Why?)



## Also Discovered

### Sophie Germain's Identity

$$\begin{aligned}x^4 + 4y^4 &= ((x + y)^2 + y^2)((x - y)^2 + y^2) \\ &= (x^2 + 2xy + 2y^2)(x^2 - 2xy + 2y^2)\end{aligned}$$

## Final Days

- Never married had no children.
- Diagnosed with breast cancer in 1829 [Cen05, p. 68]. Died June 27th, 1831.
- Death certificate lists her as a '*rentier*' (property holder) **not** 'mathematician' [OR].
- Had two primary biographers Guglielmo Libri Carucci dalla Sommaja (1803-1869) thief of ancient manuscripts) and Hippolyte Stupuy (1830-1900 French journalist)
- Libri's letters are held at the Moreniana Library of Florence [Cen05]

# Plaque

- Sophie's final home was at 13 rue de Savoie, Paris France where this plaque can be found

*Sophie Germain, philosophe et mathématicienne, ne a Paris en 1776, est morte dans cette maison le 27 juin 1831. Mars 1894. Ville de Paris.*

*Sophie Germain, philosopher and mathematician, born in Paris in 1776, died in this mansion June 27th, 1831. (March 1894, City of Paris)*



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