Enumeration and classification of single change covering designs

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Example 1

SCCD(v, k, t, b)

1	1	1	1	3	3	2
2	2	5	6	6	6	6
3	4	4	4	4	5	5

Example 1

SCCD$(6, 3, 2, 7)$

1	1	1	1	3	3	2
2	2	5	6	6	6	6
3	4	4	4	4	5	5

Example 2

$$\mathrm{SCCD}(12, 4, 2, 22)$$

0	0	0	0	7	7	3	3	3	3	3	3	8	8	10	10	10	10	10	10	2	2
1	1	1	6	6	6	6	6	10	11	11	11	11	11	11	11	11	11	8	7	7	9
2	2	5	5	5	8	8	9	9	9	9	9	9	2	2	2	1	1	1	1	1	1
3	4	4	4	4	4	4	4	4	4	7	5	5	5	5	6	6	0	0	0	0	0

Example 3

SCCD$(8, 4, 3, 19)$

0	0	0	0	0	0	0	4	5	5	5	5	5	5	5	3	3	3	3
1	1	1	1	1	3	2	2	2	2	2	0	0	0	1	1	1	1	1
2	2	2	2	7	7	7	7	7	3	3	3	7	6	6	6	7	7	7
3	4	5	6	6	6	6	6	6	6	4	4	4	4	4	4	4	5	2

Example 3

SCCD(8, 4, 3, 19)

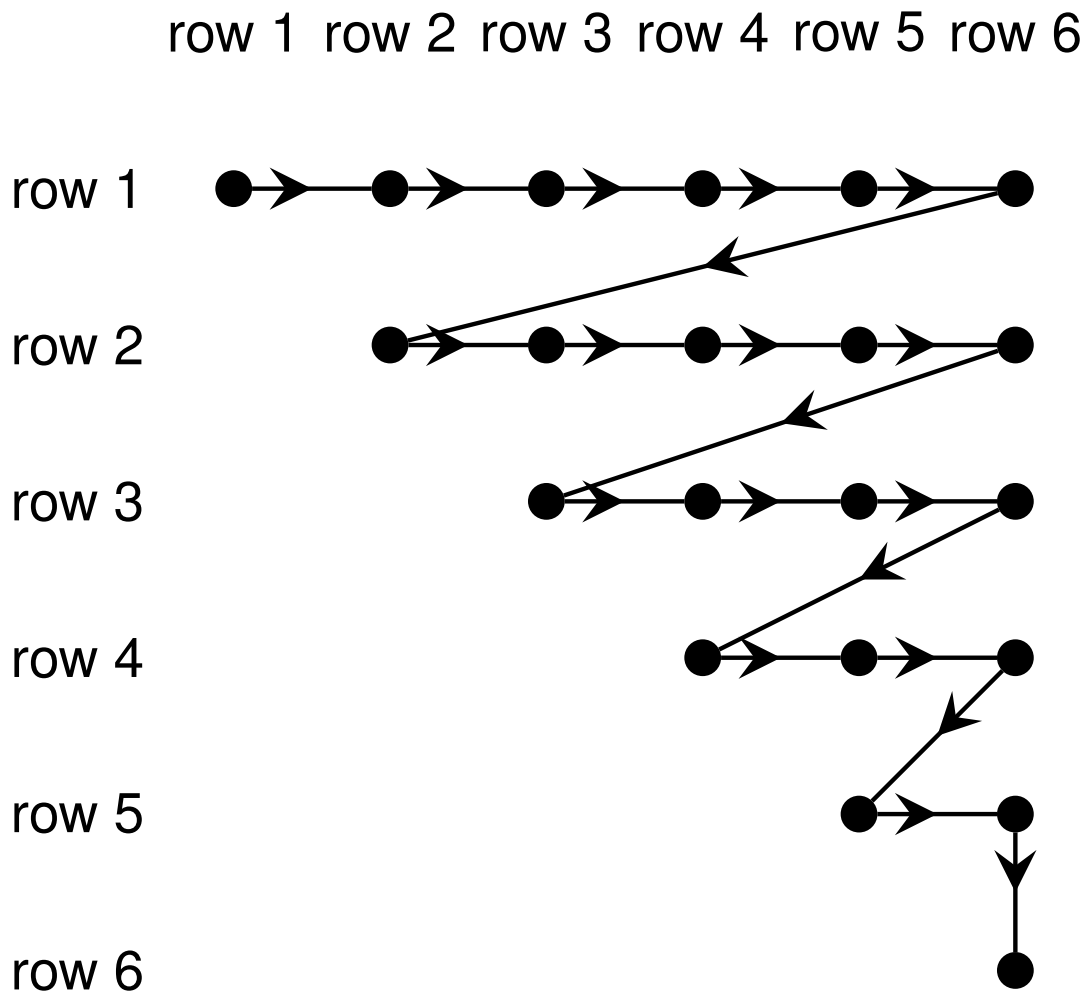
0	0	0	0	0	0	0	4	5	5	5	5	5	5	5	3	3	3	3
1	1	1	1	1	3	2	2	2	2	2	0	0	0	1	1	1	1	1
2	2	2	2	7	7	7	7	7	3	3	3	7	6	6	6	7	7	7
3	4	5	6	6	6	6	6	6	6	4	4	4	4	4	4	4	5	2

Matrix multiplication

Write a program to compute AA^T:

$$\begin{bmatrix} 0 & 8 & 2 & -2 & 3 & 3 \\ -2 & 7 & 9 & -1 & 1 & 4 \\ 3 & 4 & -5 & 7 & 2 & -3 \\ 4 & 1 & 0 & -1 & 6 & -5 \\ -3 & 0 & -1 & 3 & -5 & 6 \\ 2 & -2 & 4 & -1 & 5 & 1 \end{bmatrix} \begin{bmatrix} 0 & -2 & 3 & 4 & -3 & 2 \\ 8 & 7 & 4 & 1 & 0 & -2 \\ 2 & 9 & -5 & 0 & -1 & 4 \\ -2 & -1 & 7 & -1 & 3 & -1 \\ 3 & 1 & 2 & 6 & -5 & 5 \\ -3 & 4 & -3 & -5 & 6 & 1 \end{bmatrix}$$

Pairs visited by double loop



Consecutive states of cache

Double loop:

1	1	1	1	2	2	2	2	2	3	3	3	3	4
2	4	4	6	6	3	3	5	5	5	4	4	6	6
3	3	5	5	5	5	4	4	6	6	6	5	5	5

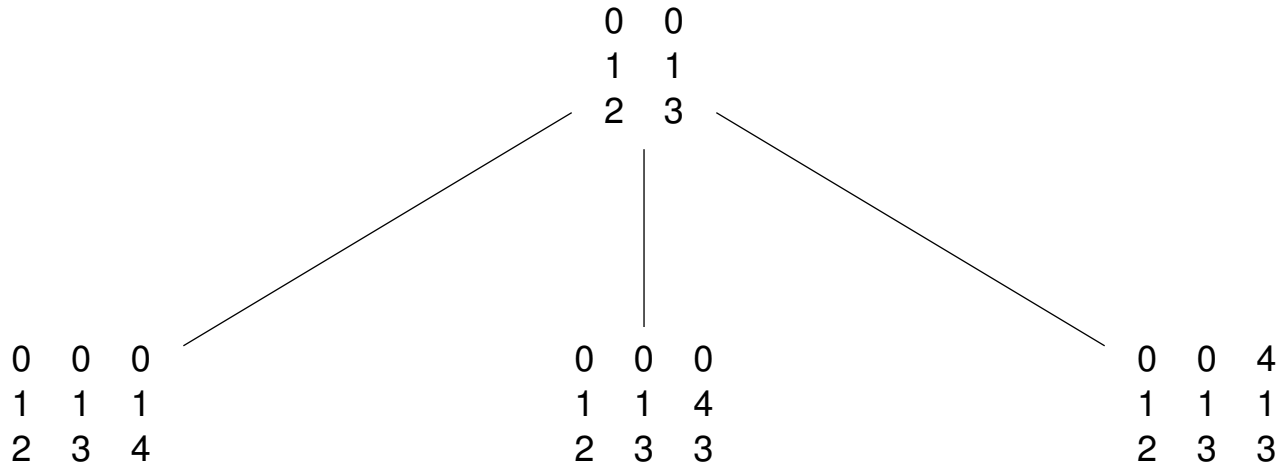
SCCD:

1	1	1	1	3	3	2
2	2	5	6	6	6	6
3	4	4	4	4	5	5

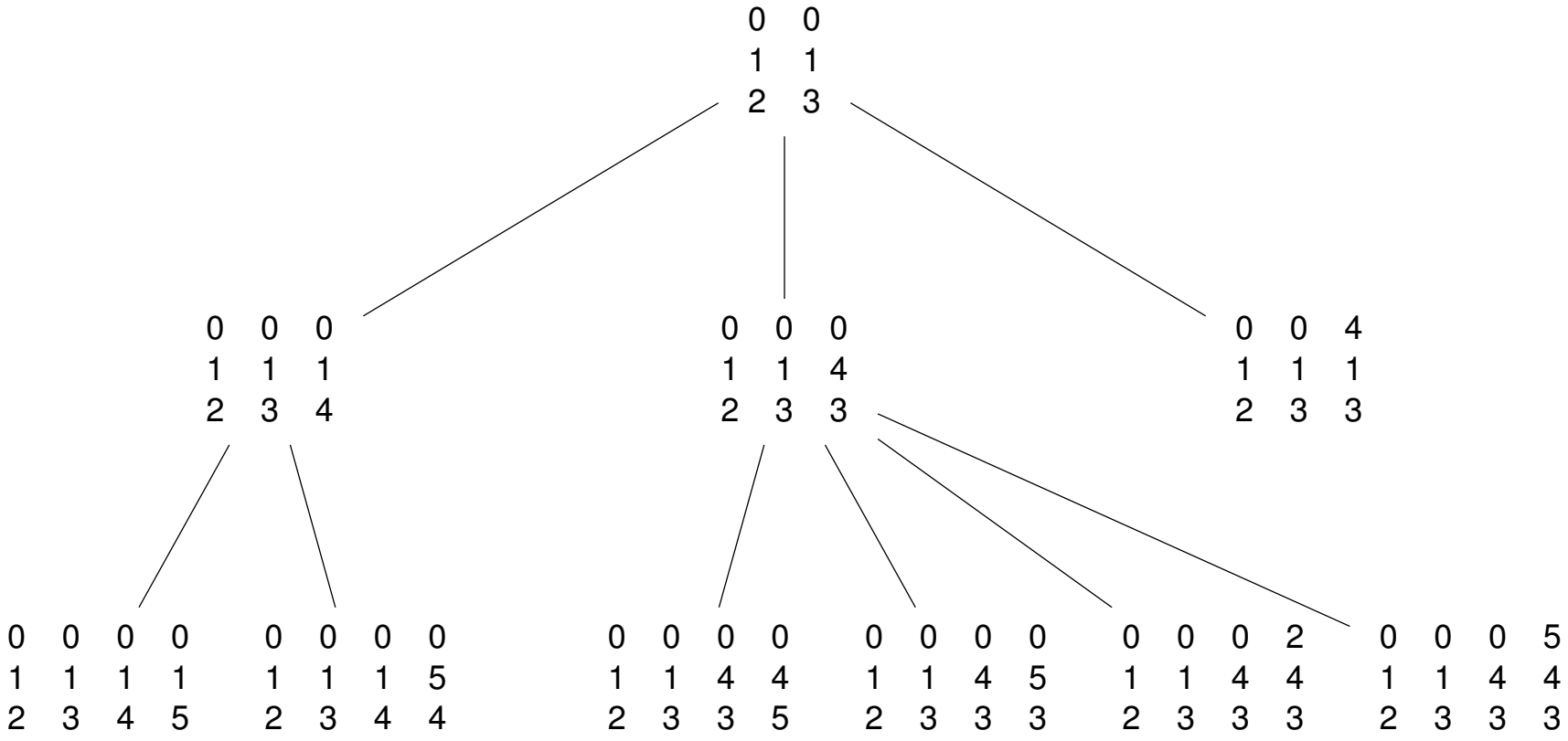
Canonical parent/augmentation

0	0
1	1
2	3

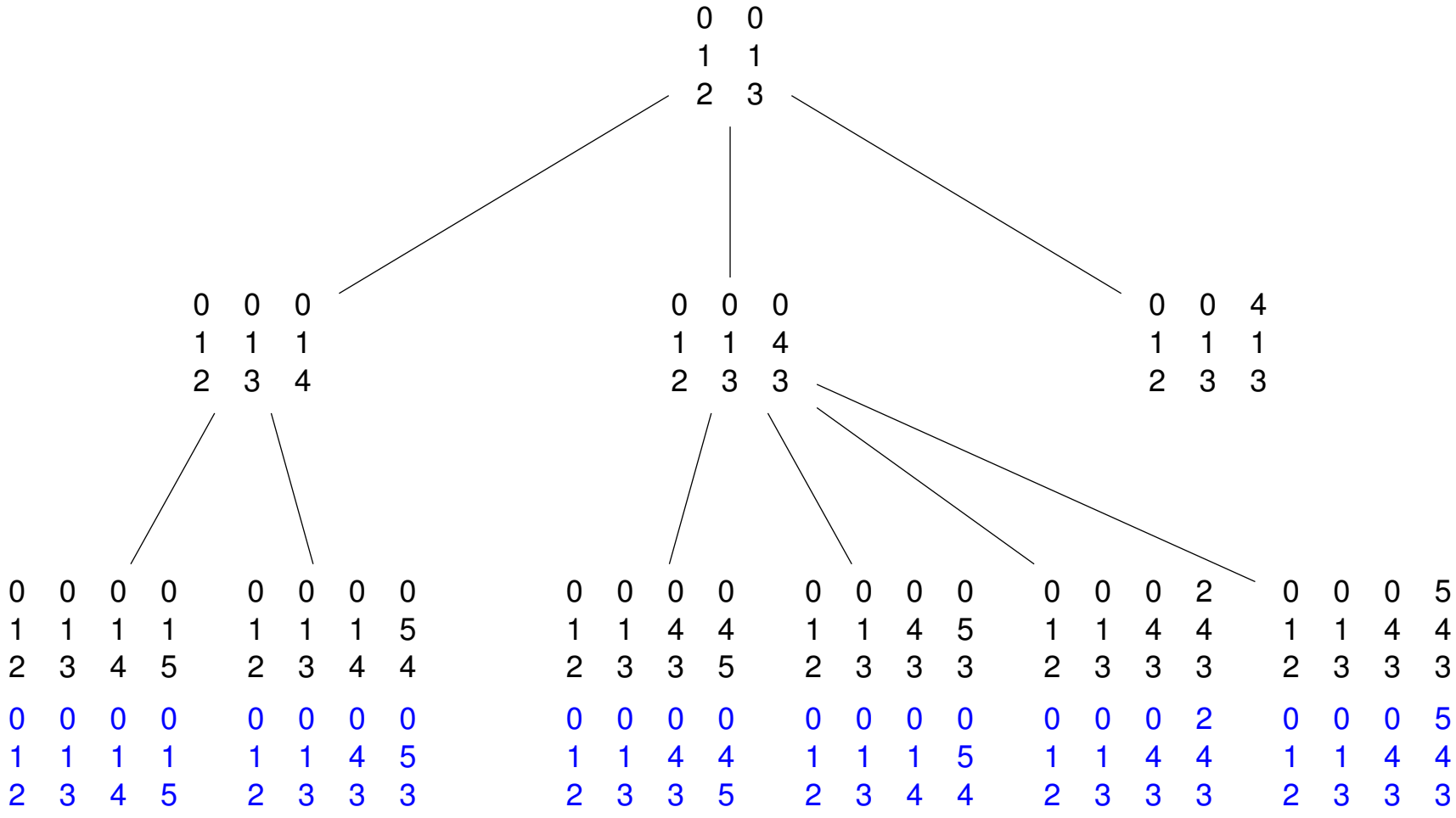
Canonical parent/augmentation



Canonical parent/augmentation



Canonical parent/augmentation



Large class reduction

1	1	1	1	1	1	3	4	4	4	2	2	1	3	7	7	6	6	6	6	6
2	2	2	2	2	9	9	9	9	9	9	12	12	12	12	5	5	5	5	5	5
3	3	3	7	7	7	7	7	10	10	10	10	10	10	10	10	10	8	8	4	9
4	5	6	6	8	8	8	8	8	11	11	11	11	11	11	11	11	11	12	12	12

Large class reduction

1	1	1	1	1	1	3	4	4	4	2	2	1	3	7	7	6	6	6	6	6
2	2	2	2	2	9	9	9	9	9	9	12	12	12	12	5	5	5	5	5	5
3	3	3	7	7	7	7	7	10	10	10	10	10	10	10	10	10	8	8	4	9
4	5	6	6	8	8	8	8	8	11	11	11	11	11	11	11	11	11	12	12	12

Large class reduction

1	1	1	1	1	1	3	4	4	4	2	2	1	3	7	7	6	6	6	6	6
2	2	2	2	2	9	9	9	9	9	9	12	12	12	12	5	5	5	5	5	5
3	3	3	7	7	7	7	7	10	10	10	10	10	10	10	10	10	8	8	4	9
4	5	6	6	8	8	8	8	8	11	11	11	11	11	11	11	11	11	12	12	12

linear $t = 2$

v	k	b	# large class	# iso class	# designs
4	3	3	1	1	1
5	3	5	1	3	5
6	3	7	1	1	2
7	3	10	6	16	32
8	3	14	198	26765	53530
5	4	3	1	1	1
6	4	4	0	0	0
7	4	6	0	0	0
8	4	9	3	32	64
9	4	11	0	0	0
10	4	14	0	0	0
11	4	18	2839	170366	340732
12	4	21	240	1277	2554

circular $t = 2$

v	k	b	# large class	# iso class	# designs
4	3	3	1	1	1
5	3	5	1	1	1
6	3	8	1	6	72
7	3	11	1	63	1342
8	3	14	9	63	1764
9	3	18	428	5233	188036
5	4	4	1	2	3
6	4	5	1	1	1
7	4	7	1	1	1
8	4	10	1	25	400
9	4	12	2	2	16
10	4	15	5	6	136
11	4	19	908	37913	1440618
12	4	22	126	654	28776

linear $t = 3$

v	k	b	# large class	# iso class	# designs
4	3	4	1	1	1
5	3	10	7	398	796
6	3	20	152,435,291–304,970,582		
5	4	3	0	0	0
6	4	7	1	3	4
7	4	12	4	4	8
8	4	19	135	867	1716
6	5	3	0	0	0
7	5	6	0	0	0
8	5	9	0	0	0
9	5	14	0	0	0
10	5	20	0	0	0

circular $t = 3$

v	k	b	# large class	# iso class	# designs
4	3	4	1	1	1
5	3	10	2	31	532
6	3	20			$\leq 19,005,592$
5	4	4	1	1	1
6	4	7	0	0	0
7	4	12	0	0	0
8	4	19	1	2	76
6	5	4	1	1	1
7	5	6	0	0	0
8	5	10	0	0	0
9	5	14	0	0	0
10	5	20	0	0	0

Moving forward

- Find $t = 3$ ingredients for recursive construction
- Move from OpenMP to MPI
- Metaheuristics
- SCCA
- Use Canonical parent/augmentation on other problems

Marsee