Unsatisfiability Proofs for Weight 16 Codewords in Lam's Problem

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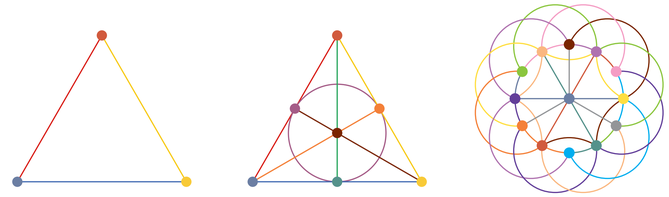
Motivation

Many mathematical problems concern the existence of combinatorial objects that are only feasibly constructed through a search. For example, **Lam's problem**—determining if a projective plane of order ten exists—was studied since the 1800s and only resolved via a supercomputer search in the 1980s.

Finite Projective Planes



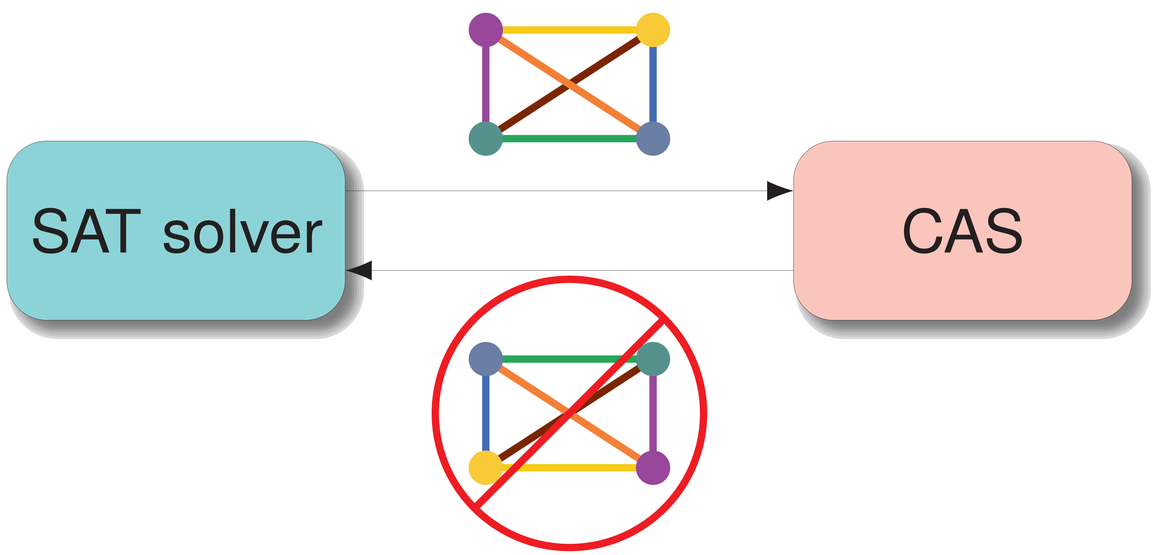
Every pair of lines meet at a unique point.
There is a unique line through any two points.
Every line contains $n+1$ points (in order n).

Results

We reduce Lam's problem to Boolean logic and use SAT solvers and computer algebra systems to generate the first collection of nonexistence certificates for the problem. A subcase of Lam's problem that was previously solved in 16,000 computing hours was resolved by our system in 30 hours.

The MathCheck SAT+CAS System

A satisfiability (**SAT**) solver finds partial projective planes…



…and a computer algebra system (**CAS**) finds nontrival isomorphisms and blocks them.