A SAT+CAS Approach to Finding Good Matrices

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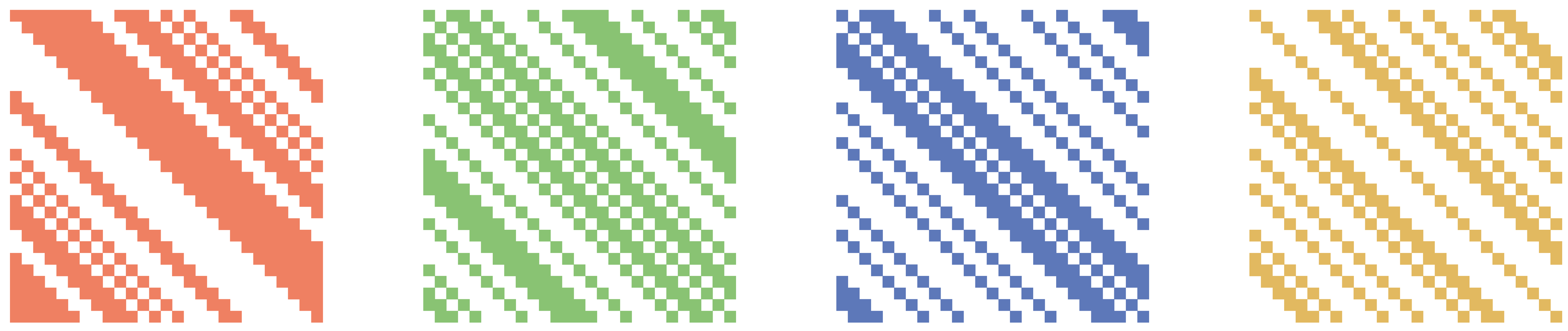
Motivation

Many mathematical conjectures concern the existence of combinatorial objects that are only feasibly constructed using a clever search procedure such as those used by SAT solvers.

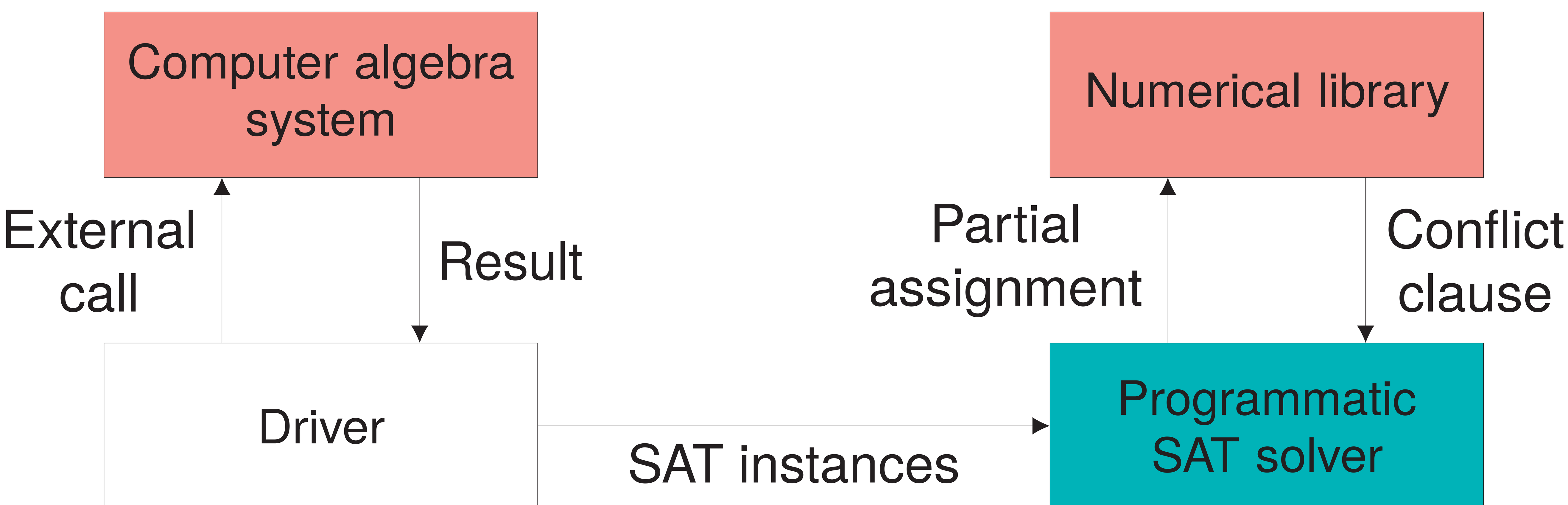
Good matrices

Four $n \times n$ matrices A, B, C, D with ± 1 entries, B, C, D symmetric, $A - I$ anti-symmetric, each row a shift of the previous row, and $AA^T + BB^T + CC^T + DD^T = 4nI$.



New good matrices A, B, C, D in order 27.

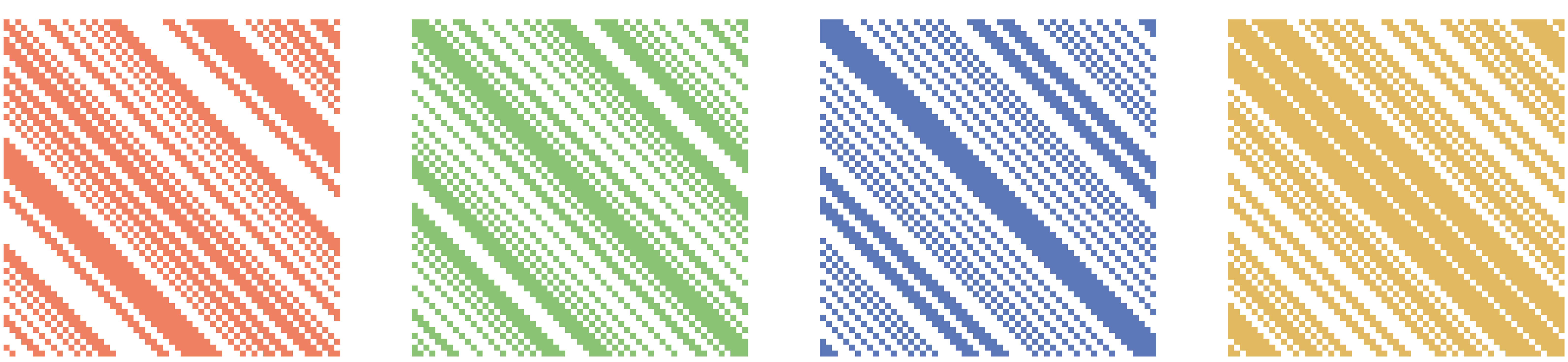
The MathCheck SAT+CAS System



With the help of an external CAS a script splits the search for good matrices of order n into subspaces and generates a SAT instance for each subspace. Some instances can be ruled out because they encode subspaces which violate theorems that good matrices are known to satisfy.

A SAT solver performs an exhaustive search through each subspace. Periodically it will compute the Fourier "power spectra" of the current assignment. If the spectra is not consistent with the spectra of good matrices then a clause is learned blocking the current assignment.

Conjecture: Good matrices exist in all odd orders.
New Counterexamples: There are no good matrices in orders 51, 63, and 69.
New Example: There are good matrices in order 57.



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