A SAT-based Resolution of Lam's Problem

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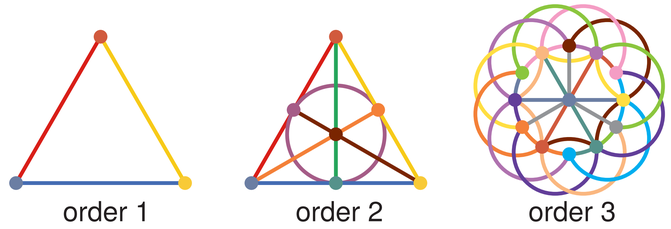
Motivation

Many mathematical problems concern the existence of combinatorial objects that are only feasibly constructed through a search. For example, **Lam's problem**—determining if a projective plane of order ten exists—was studied since the 1800s and only resolved via a supercomputer search in the 1980s.

Finite Projective Planes



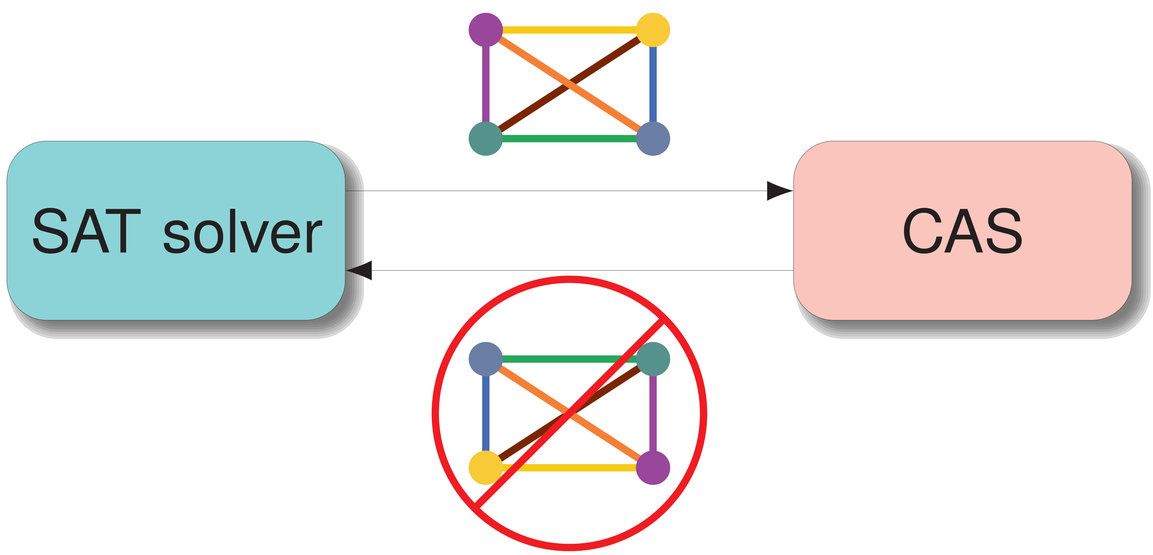
Every pair of lines meet at a unique point.
There is a unique line through any two points.
Every line contains $n+1$ points (in order n).

Results

We solve Lam's problem by reducing it to Boolean logic. Using SAT solvers and computer algebra systems we generate the first set of nonexistence certificates for the problem and provide the fastest known solution—solving one subcase 500 times faster when compared with a verification from 2011.

The MathCheck SAT+CAS System

A satisfiability (**SAT**) solver finds partial projective planes...



...and a computer algebra system (**CAS**) finds nontrival isomorphisms and blocks them.