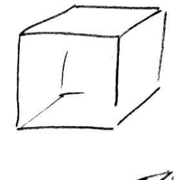
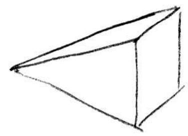
3/25/10 Lecture 20: Turaev-Viro codes

project

Last time  for $|0\rangle, |+\rangle$ preparation, CNOT in toric code

 for encoding a magic state

Goals: 1. Geometrically local codes with easier universality, similar threshold?
2. Realization of (nearly) arbitrary anyon models from a qubit lattice.

Map: 1. Anyon model (tensor category)
2. Desired microscopic behavior
3. Derive macroscopic anyons
4. Implement micro features w/ local checks
5. Computation
6. Relate to TV & WRT topological invariants

[1002.2816]

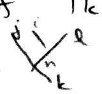
Stable qu states of matter, loop qu gravity, FT & physics

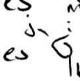
1. Anyon model: Unitary modular tensor category (without multiplicities and w/ self-dual particles)

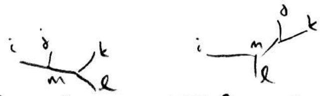
Particle types $\{0, i, j, k, \dots\}$

Qu dimensions d_i, $d_0 = 1$

Fusion rules $\delta_{ijk} = 1$ if  allowed

F-moves  $= \sum_n F^{ijm}_{kln}$ 

R-moves  $= R^{ij}_k$ 

Consistency conditions: 

associativity of fusion $\sum_m \delta_{ijm}\delta_{mkl} = \sum_m \delta_{jkm}\delta_{iml}$

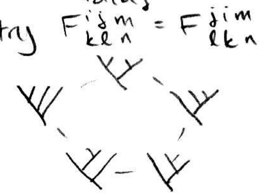
$d_i d_j = \sum_k \delta_{ijk} d_k$

physicality $F^{ijm}_{kln} = F^{ijm}_{kln}\delta_{ijm}\delta_{klm}\delta_{iln}\delta_{jkn}$

unitarity $F^{ijm}_{kln} = (F^{ijm}_{kln})^*$

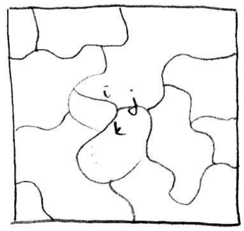
normalization $F^{ii0}_{jjk} = \sqrt{\frac{d_k}{d_i d_j}}\,\delta_{ijk}$

tetrahedral symmetry $F^{ijm}_{kln} = F^{jim}_{lkn} = F^{lkm}_{jin} = F^{imj}_{knl}\sqrt{\frac{d_m d_n}{d_j d_l}}$

pentagon equation  hexagon equation

a b c

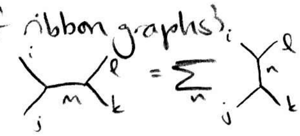
2. Microscopic behavior: ribbon-graph (string-net) condensate



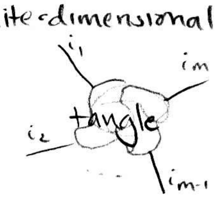
Σ = compact, orientable surface with boundary

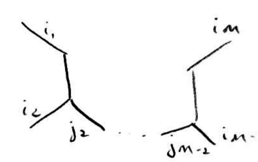
e.g. torus  or $(n+1)$-punctured sphere $\underbrace{\circ \;\circ\; \circ\; \circ}_{n}$

$\mathcal{H}_\Sigma$ (code space) = {formal linear combinations of ribbon graphs}$_i$

modulo isotopy,  $= \delta_{i0} \cdot d_j$, 

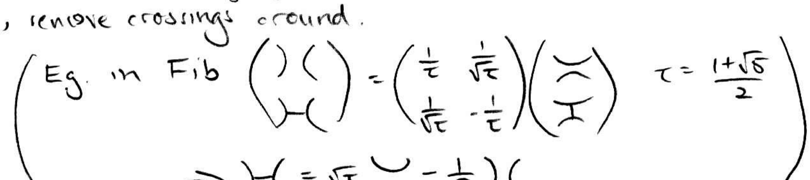
Exercise: Prove that $\mathcal{H}_\Sigma$ is finite-dimensional.

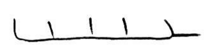
Idea: First show that any 

can be written as a linear combination of diagrams  Mac Lane

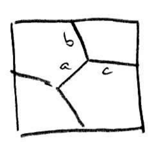
$\Rightarrow \dim \mathcal{H}^{(i_1 \dots i_m)}_{\Sigma_1} = \sum_{j_2 \cdots j_{m-1}} \delta_{i_1 i_2 j_2} \delta_{i_3 j_2 j_3} \cdots \delta_{i_{m-1} j_{m-2} i_m}$

Next, remove crossings around.



"Computational" basis states for $\mathcal{H}_{\Sigma_{n+1}}$ 

all orthonormal

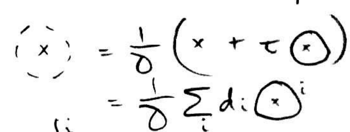
trace inner product for general Σ 

a	b	c
0	0	0
1	1	0
1	0	1
0	1	1
1	1	1

not orthonormal (4D)

3. Macroscopic anyons

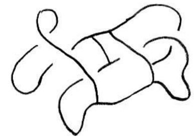
$n=1$ punctures:  Fib

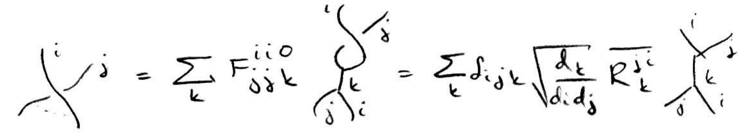
"vacuum loop" 

Claim: 

Proof:  ✓ □

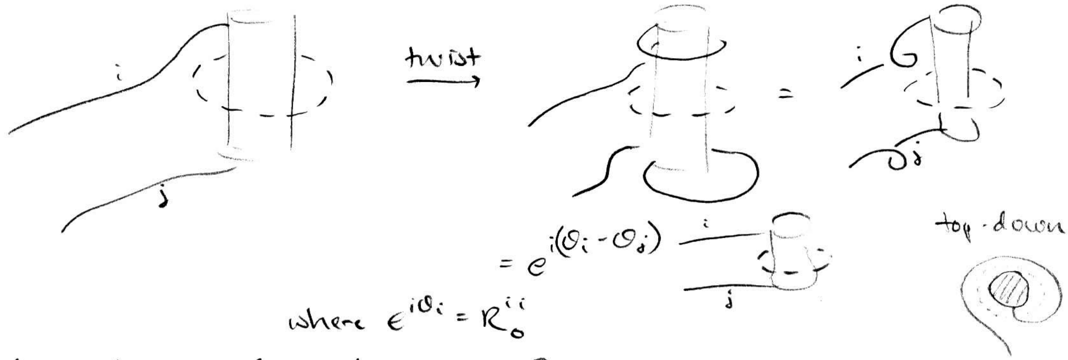
3D to 2D reduction





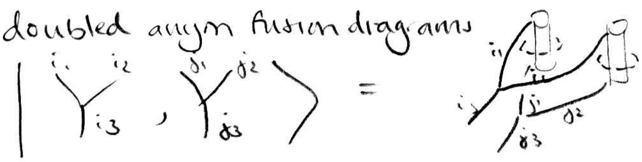
$$= \sum_k F^{ii0}_{jjk} \quad = \sum_k \delta_{ijk}\sqrt{\frac{d_k}{d_i d_j}}\, R^{ji}_k$$

to eliminate crossings



where $e^{i\theta_i} = R^{ii}_0$

$\Rightarrow$ doubled anyon fusion diagrams

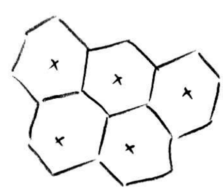


satisfy ~~roots~~ twists, F moves, braiding

$\Rightarrow$ doubled category realized

pants decomposition (boundary conditions)

4. Implement micro features with local checks



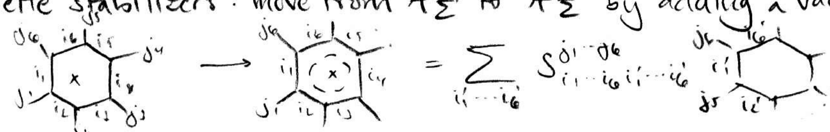
begin with a trivalent lattice, a qudit on every edge

let $\Sigma' = \Sigma$ with a puncture through every plaquette

then $\mathcal{H}_{\Sigma'}$ can be naturally embedded in the qudit Hilbert space

vertex stabilizers enforce fusion constraints $S_v = \sum_{ijk} \delta_{ijk} |ijk\rangle\langle ijk|$

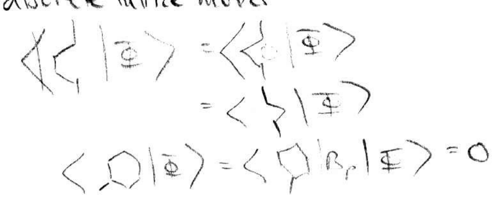
plaquette stabilizers: move from $\mathcal{H}_{\Sigma'}$ to $\mathcal{H}_\Sigma$ by adding a vacuum loop



$$= \sum_{i_1' \cdots i_6'} S^{j_1 \cdots j_6}_{i_1 \cdots i_6 i_1' \cdots i_6'}$$

5. Computation by F-moves

discrete lattice moves



6. Relationship to topological invariants

tensors and tensor-network contraction

Turaev-Viro invariant