3/23/10 Lecture 19

Fault-tolerant quantum computation with the toric code

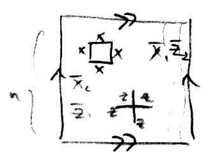
References: Toric code [Kitaev 9707021], Memory [Dennis, K., Landahl, Preskill] 0110143

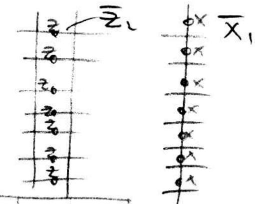
Toric code with bdry [Bravyi & Kitaev 9811052] [Freedman & Meyer] 9810055

Computation [Raussendorf Harrington Goyal 0703143] 0510135

also [0610082, 0805.3202]

<u>Recall</u> Toric code

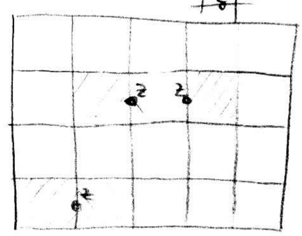




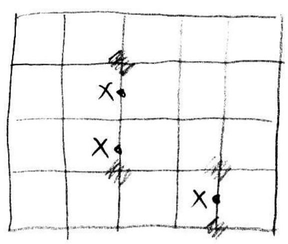
$[[n^2, 2, n]]$

Error correction:

1. with perfect syndrome extraction

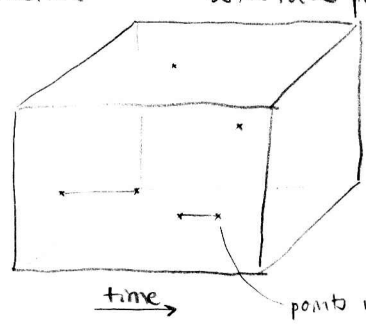


Phase error flips the adjacent two plaquettes — finding ℓ_1-shortest matching



X error flips the adjacent two stars

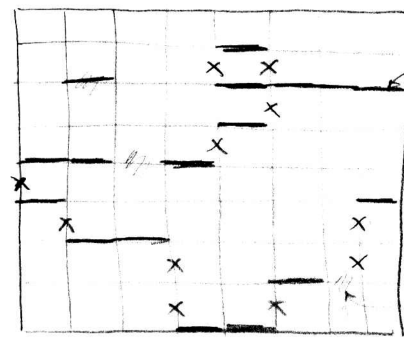
2. with imperfect syndrome extraction — both false positives & negatives



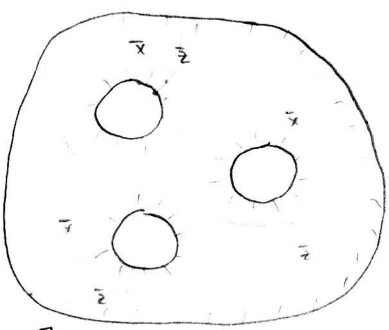
again, find a matching, now so as to minimize the ℓ_1 lengths with a different weighting in time (syndrome extraction errors) versus space (real errors)

a more sophisticated algorithm would deal with correlations btwn syndrome & real errors, and btwn X & Z errors.

Example:



Toric code with boundary



logical operators ($g=0, p=4$)

Euler: $v - e + f = 2 - 2g - \#\text{punctures}$

(trivial proof)

$\#$ qubits $= e$

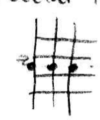
$\#$ ind. Z stabilizers $= v-1$

$\#$ ind X stabilizers $= \begin{cases} f-1 & \text{if no punctures} \\ f & \text{otherwise} \end{cases}$

$\Rightarrow \#$ encoded qubits $= e-(v-1)-f+\chi_{p=0}$

$= -1 + 2g + p + \chi_{p=0}$

Notice: A chain of Zs on the dual lattice can end at a smooth bdry w/o flipping any syndromes



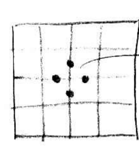
Z half loops

are in the stabilizer

for syndrome extraction

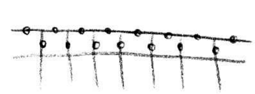
Initializing a qubit:

1. By cutting a hole:



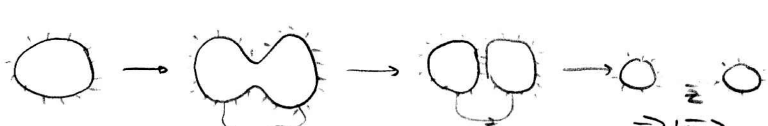
→ measure Z
- destroys 3 plaquettes + star stabilizer
- leaves $\bar{X}$ $\Rightarrow |+\rangle$

- then expand the hole



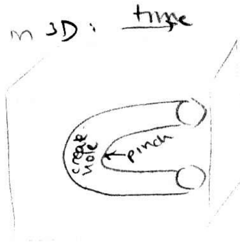
→ measure Z

2. By splitting a hole

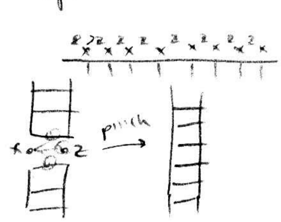


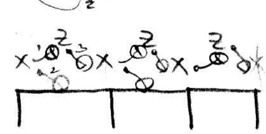
$\Rightarrow |\bar{0}\rangle$

in 3D: time

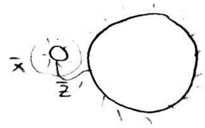


basic operation:





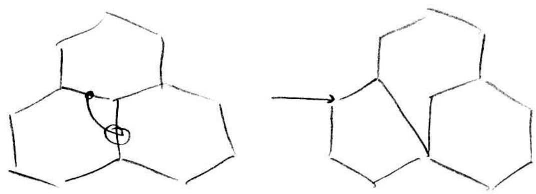
to expand smooth bdry

3. 

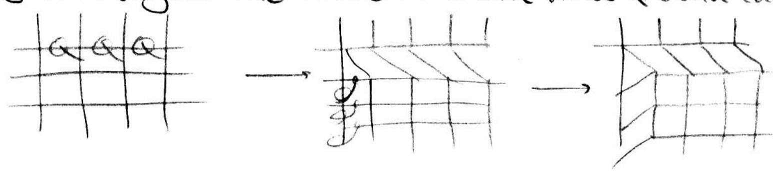
If one side of the pinch is small enough, can create arbitrary state, then enlarge and separate the holes to stabilize the state (leaves a constant noise)

<u>Computation</u>:

Notice

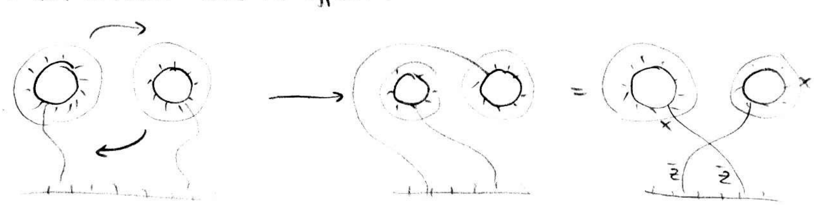


Thus can adjust the lattice to braid holes around each other

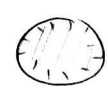


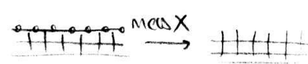
<u>Or</u>: Can simply grow holes from one side, shrink from the other

But such braids have no effect?

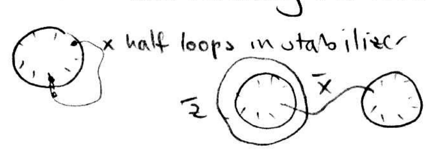


Partial solution:

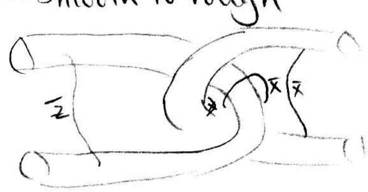
Allow holes with rough boundaries 

-dimension-counting is the same as before 

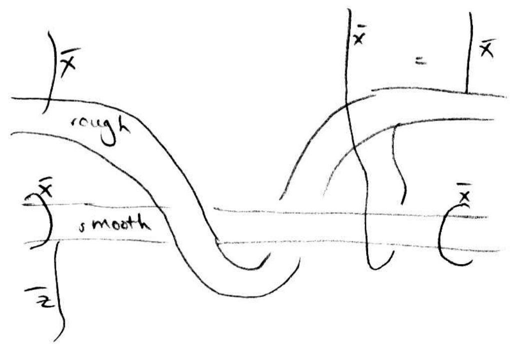
eliminates as many stabilizers as qubits



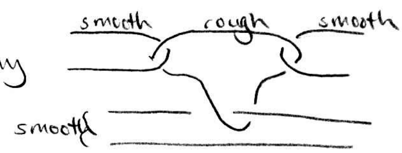
Conversion smooth to rough



→



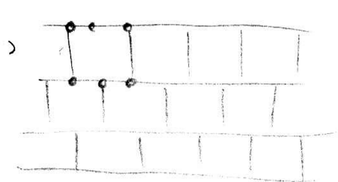
$\Rightarrow$ for a CNOT, apply



measurement + magic states distillation gives universality
threshold determinant

Generalizations of the toric code:

1. Quantum double models — generalized Clifford group, non-abelian case eg. [0901.1345]
(2. Color codes) [0605138]
3. Turaev-Viro codes [1002.2816]



qubits on vertices
for each plaquette $\bigotimes_{v\in p} X_v$, $\bigotimes_{v\in p} Z_v$