CS 798 Lecture 16 3/11/10 Toric code
Ising model, project

Anyons

review article Nayak et al. 0707.1889

cover anyon models, many proposed implementations

compiling gates into braids: Simon et al. 0509175

one mobile quasiparticle

(note: can use threshold thm instead of Solovay Kitaev)

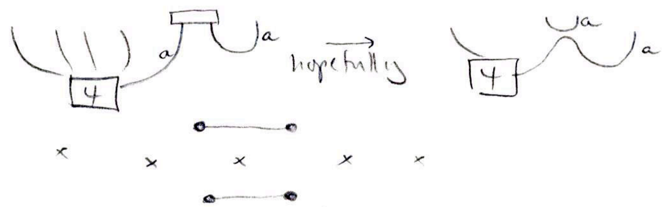
BFN 0808.1933

FQH model, anyon density matrices & partial traces,

projective & interferometric measurements, error analysis

measurement-only TQC

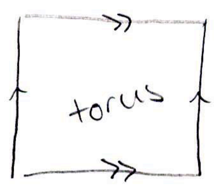
rough idea for anyonic teleportation

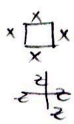


Ising model, magic states distillation, composite anyon distillation

Toric code Kitaev 9707021 (first TQC paper)

motivation: realizing anyons on a lattice
FT q.c.

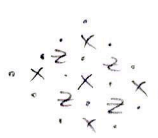




for every plaquette (tile)

for every vertex

(dual lattice)



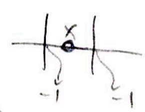
$2n^2$ physical qubits

$2(n^2-1)$ independent stabilizers

$\Rightarrow 2$ qubits (where?)

constructing logical X operators.

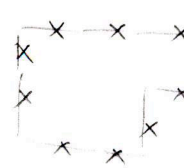
begin with an X error



lengthening the error, syndromes

show up at the endpoints

$\Rightarrow$ closed cycles are undetectable

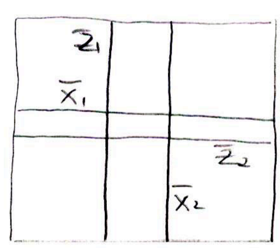


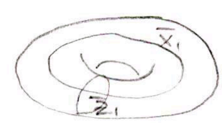
$\in S$

cycle around torus $\in N(S) \setminus S$

similarly for Z

$\Rightarrow$ distance $= n$





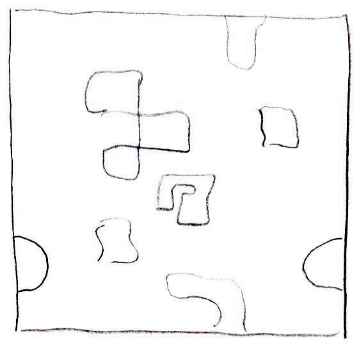
$\therefore$ transpose implements swap

transversal H implements $-H-$ $-H-$

codewords:

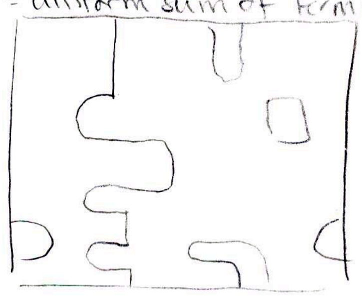
$$|\bar{0}\bar{0}\rangle \propto \prod_{\text{plaquettes}} \left(I + \prod_{e \in \square} X_e\right) |0^{2n^2}\rangle$$

= uniform superposition of all sets of contractible cycles



"strong condensate"

$|\bar{0}\bar{1}\rangle$ = uniform sum of terms



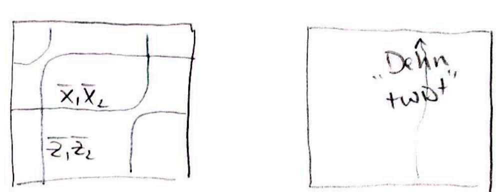
initialization, measurement

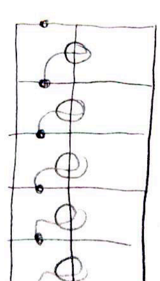
ex: log-depth encoder

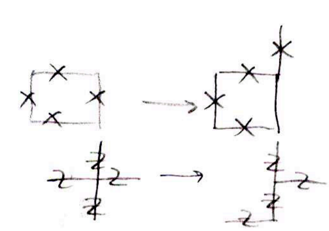
logical CNOT within the code by code deformation

$X_1 \to X_1 X_2$

$Z_2 \to Z_1 Z_2$





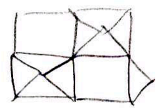
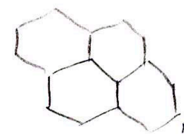


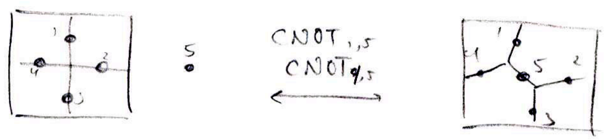
repeat n times, with error correction in between

→

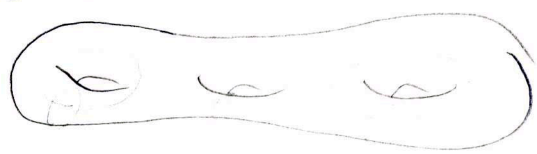
Generalizations:

1. non-square plaquettes

eg.  or  ↖ but stabilizers are less local



2. higher-genus surfaces



Euler $v - e + f = 2 - 2g$ [$-$ # punctures]

e ↑ # qubits

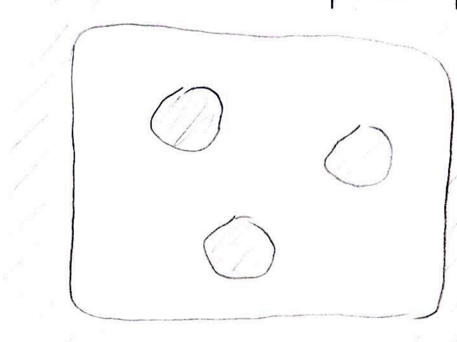
indep Z stabilizers $= v - 1$

indep X stabilizers $= f - 1$

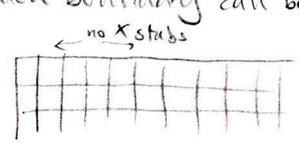
⟹ # encoded qubits $= e - (v-1) - (f-1)$

$= 2g$

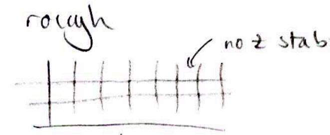
3. Toric code on the plane (punctured sphere)



each boundary can be smooth



or rough



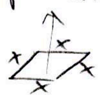
chains of Zs can end on a smooth bdry

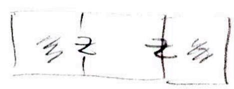
chains of Xs on a rough bdry

ex: derive code parameters, logical operators

Error correction: by matching & by renormalization

Interpretation in terms of abelian anyons

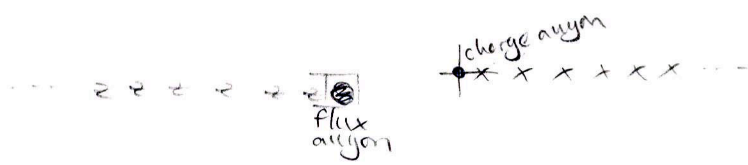
 = magnetic field $\mathbb{Z}/2\mathbb{Z}$ = charge conservation



"magnetic vortex" = face w/ syndrome -1
"electric charge" = vertex " "

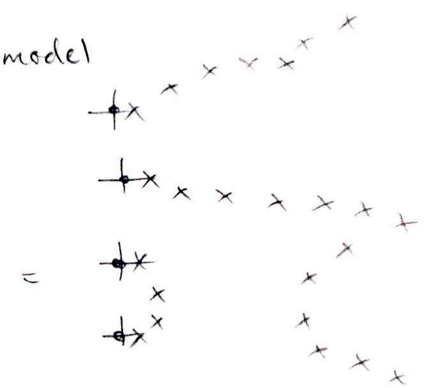
pairs of vortices created by strings of Z errors on the dual lattice
pairs of charges created by X error strings
⇒ no single particles (no longer true w/ bdrys)

braiding statistics



flux w/ flux +1
charge w/ charge +1
flux w/ charge -1

$\mathbb{Z}_2 \otimes \mathbb{Z}_2$ model



(0,0) trivial   (1,0) flux   (0,1) charge   (1,1) flux+charge

2/24/10 Simulating $SU(2)_2$ using Clifford gates

I think it is known that the Ising model, and therefore presumably also $SU(2)_2$, can be simulated efficiently using classical processing and Clifford gates. Let me see if I can rederive this.

$\times$	0	$1/2$	1
0	0	$1/2$	1
$1/2$	$1/2$	$0+1$	$1/2$
1	1	$1/2$	0

Consider a set of n $1/2$ anyons together with 0 and 1 anyons. We will store the structure of the fusion tree in our classical memory, and use $\lfloor \frac{n}{2} \rfloor$ qubits for the anyon labels that can be 0 or 1.

This only happens when two $1/2$ anyons come together. Thus the label of an edge in the tree can be determined as follows:

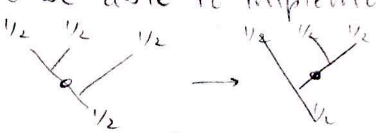
- If the parity of the number of $1/2$ anyons above it is odd, then it is $1/2$.
- Otherwise, go up to the nearest places where $1/2$ anyons are fusing. This edge's label is the parity of all those labels, i.e., a $Z \otimes Z \otimes \cdots \otimes Z$ operator.

For example

← this edge label is a $Z \otimes Z$ on the two qubits above.

We need to be able to implement braids and F-moves.

* For F-moves:



is given by $\frac{1}{\sqrt{2}}\begin{pmatrix} -1 & 1 \\ 1 & 1 \end{pmatrix}$, a Clifford.

In all other cases, the fusion space is 1-dimensional. And no phase is introduced except for a -1 when there are two 1 and two $1/2$ anyons. Thus when there are two $1/2$ anyons, and two anyons that can be 0 or 1 coming in, implement the F move by applying $\Lambda(Z)$ between the $0/1$ anyons. (That is, use CNOTs to compute each label as a parity, apply $\Lambda(Z)$, then uncompute the labels.)

* For R braids: $R_0^{\frac12 \frac12} = e^{5\pi i/8}$, $R_1^{\frac12 \frac12} = e^{i\pi/8}$ ⇒ braiding two $1/2$ anyons can be implemented using $\begin{pmatrix} 1 & 0 \\ 0 & -i \end{pmatrix}$ gate.

$R_0^{00} = R_1^{10} = R_1^{01} = 1$, $R_0^{11} = -1$ ⇒ compute parities on both sides, apply

this is +1 for Ising

$$\begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & -1 \end{pmatrix} = \Lambda(Z) \checkmark$$

$R_{1/2}^{0\,1/2} = 1$, $R_{1/2}^{1\,1/2} = i$ ⇒ apply $\begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix} \checkmark$

$R_{1/2}^{1/2\,0} = 1$, $R_{1/2}^{1/2\,1} = i$ ⇒ $\begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix} \checkmark$

Converse.