Noise threshold overview · Upper bounds, Existence proofs, Lower bds, Simulations

Threshold upper bounds

Aharonov, Ben-Or, Impagliazzo, Nisan 9611028

for more than log depth (QNC'), need fresh source of ancillas

$I(\rho) := n - S(\rho) \xrightarrow[\text{indep. depolarizing}(p)]{} (1-\frac{4}{3}p) I(\rho)$

refridgeration

(obvious): need full parallelism, no global noise or growing noise w/ size of system

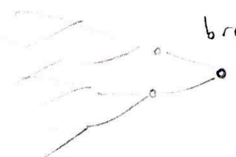
offset by good memory

Aharonov, Ben-Or 96

for fan-in 2 gates, if $\frac{4}{3}p > 0.97 \Rightarrow$ log depth

Razborov 0310136

if $\frac{4}{3}p > 1 - \frac{1}{k}$ for fan-in k gates (arbitrary fan-out) $\Rightarrow$ log depth



Kempe, Regev, Unger, de Wolf 0802.1464

ditto for $k=2$, $\frac{4}{3}p > 35.7\%$ (or CNOT and $\frac{4}{3}p > 29.7\%$)

Pauli decomposition $\rho = \sum_P \left(\frac{1}{2^n} \mathrm{Tr}(P\rho)\right) P$

$\xrightarrow{\mathcal{E}_i(p)} \left(\sum_{P: i \notin P} \hat{\rho}_P P + \sum_{P: i \in P} (1-\frac{4}{3}p) \hat{\rho}_P P \right)$

all for no classical control

Heuristic bounds

Bruss et al 98: capacity of depolarizing channel to transmit qu. information is 0 for depol rate $\frac{4}{3}p \geq \frac{1}{3}$.

Plenio & Virmani 0810.4340 (also Knill): bounds for particular magic-states-distillation-based schemes

Bounds allowing classical control

Harrow & Nielsen 0301108

multiqubit gates seperability preserving $\Rightarrow$ classically simulable (any depth)

CNOT w/ indep. depolarizing noise $\frac{4}{3}p \geq 0.74$

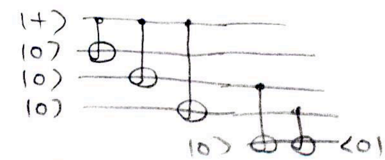
CNOT w/ adversarial rate-p noise $p \geq 0.5$

magic-states/Gottesman-Knill bounds

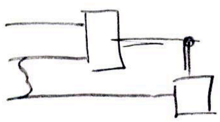
Existence proofs:

1. Leakage errors $|0\rangle, |1\rangle, |2\rangle, |3\rangle, \ldots$

computational space   leaks

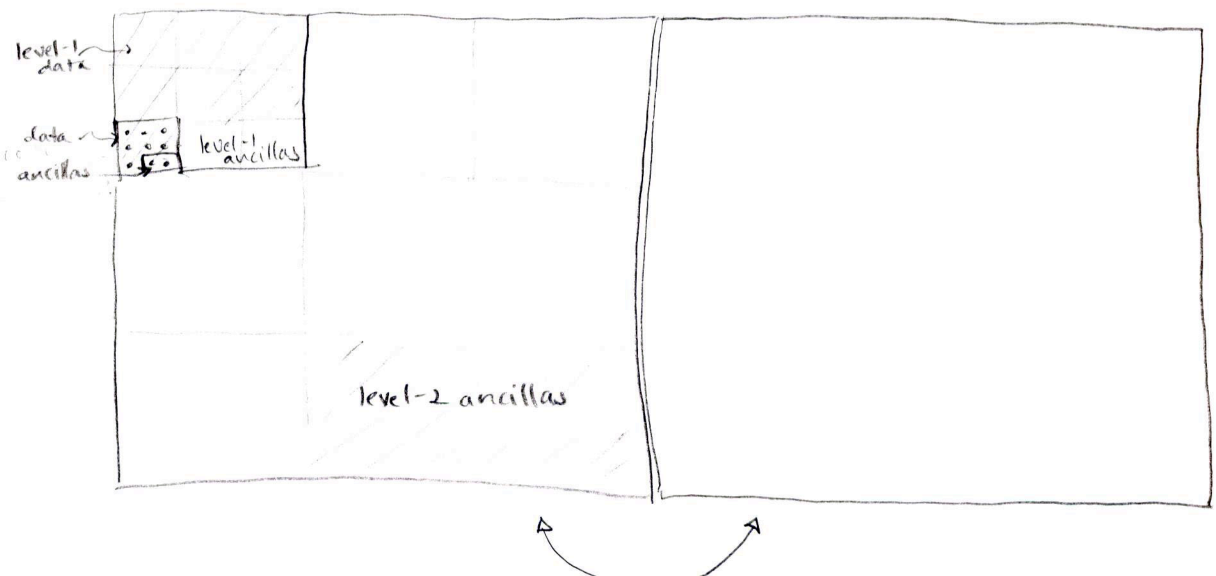


fault-tolerant against X errors
but CNOT on leaks is undefined



teleportation eliminates leakage
(free with Knill-type error correction)

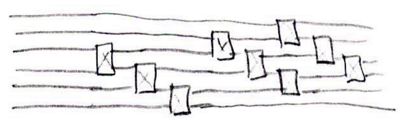
2. Locality constraints   [Gottesman 9903099]   models: grid of qubits, or grid plus paths



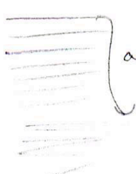
level-2 swap, implemented by level-1 swaps, with level-1 error correction (uses level-0 swaps)

Is this fault-tolerant?

No.



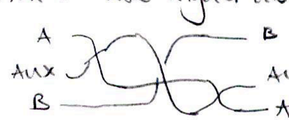
1. Leakage errors may propagate badly (fix as above)
2. Leads to correlated errors w/in block



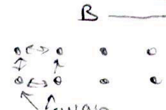
as this qubit moves down, get weight-2 errors w/ first-order probability

possible solution 1: use higher distance code

solution 2:



using next-nearest nbor gates in 1D

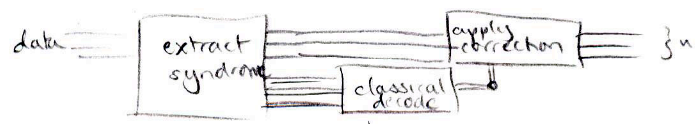


← Aux qubits
← data qubits   in 1D × 2

solution 3: change the model to allow paths

3. All unitary control

Issue:



assume reversible

(note: corrections need to be applied before any non-Clifford gate)

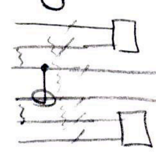
not fault-tolerant to classical errors

Trivial solution: repeat full circuit n times, determining the correction for each data qubit independently.

(Fault-tolerant classical control, using the repetition code, phase errors irrelevant)

4. Further extensions

Knill-style computation + error correction



no exRecs ~~AGP~~

different techniques can apply R?

also simulations

Thresholds for non-Markovian noise

$H = H_S + H_B + H_{SB}$

1. if S/B only interacts touching data qubits

$$\varepsilon = \max \| H_{SB}(t) \| \cdot t_0$$

2. for noise coupling all data qubits, decaying in space

$$\varepsilon_*^2 = \max_i \sum_j \| H_{ijB} \| \cdot t_0$$

b/c single insertion can break two gates (not strictly fault tolerant)

– norms are not measurable, may be ∞

e.g. bath a harmonic oscillator $H = a^\dagger a$

$|0\rangle, |1\rangle, |2\rangle, \ldots \qquad a^\dagger |j\rangle = |j+1\rangle \cdot \sqrt{j} \qquad a^\dagger = \begin{pmatrix} 0 & & & \\ \sqrt{1} & 0 & & \\ & \sqrt{2} & 0 & \\ & & \sqrt{3} & 0 \\ & & & & \ddots \end{pmatrix}$

intuition whether bath can reach high-energy space

unclear [Aliferis et al] typical coupling $Z_{\text{qubit}} \otimes (a + a^\dagger)_{\text{bath}}\, \varepsilon$

partial progress [Ng & Preskill, 0810.4953]

Lower bounds for specific fault-tolerance schemes, noise models

depolarizing noise
biased noise Pauli
} ⇒ adversarial noise at level 1

postselection + Fibonacci schemes

Knill 0402171, 0404104, <u>0410199</u> Nature

R

AP 0809.5063

Simulations

Steane 0207119

AGP-style simulations & estimates