ECE725/CS745 Winter 2011
Homework 1

(Propositional and Predicate Logics)

1. A prisoner has two guards. One is always honest and the other always lies. The prisoner does not know which one is honest and which one is the liar. There are two gates in the prison: one goes to freedom, another goes to sudden and even immediate death! Both guards know which gate goes to where. How can the prisoner ensure freedom by asking one and only one question from one guard? Support your answer using propositional logic.

2. Let $p \overline{\vee} q$ denote the connective with the following semantics:

p	q	$p \overline{\vee} q$
T	**T**	**F**
T	**F**	**F**
F	**T**	**F**
F	**F**	**T**

   Show that all binary propositional connectives (i.e., $\wedge, \vee, \Rightarrow, \Leftrightarrow$) can be defined using $\overline{\vee}$.

3. Show that connectives $\wedge, \vee, \Rightarrow$ cannot be defined using connectives $\neg$ and $\Leftrightarrow$ only.

4. Show that $\hookrightarrow (A \Rightarrow (B \Rightarrow C)) \Leftrightarrow ((A \Rightarrow B) \Rightarrow (A \Rightarrow C))$ in sequent calculus.

5. Let S be a set of formulas. An *associated sequent* for S is a sequent $\Gamma \hookrightarrow \neg\Delta$, where Γ, Δ is a partition of S (i.e., $\Gamma \cap \Delta = \emptyset$ and $\Gamma \cup \Delta = S$), and $\neg\Delta$ denotes the set $\{\neg X \mid X \in \Delta\}$. Show that if an associated sequent for S has a proof, then every associated proof does.

6. Prove or disprove the following derivations:

   - $\forall x \bullet \exists y \bullet P(x, y) \vdash \exists y \bullet \forall x \bullet P(x, y)$
   - $(\forall x \bullet P(x) \wedge Q(x)) \vdash ((\forall x \bullet P(x)) \wedge (\forall x \bullet Q(x)))$

7. Show that

$$(\forall x \bullet P(x) \vee Q(x)) \Leftrightarrow ((\forall x \bullet P(x)) \vee (\forall x \bullet Q(x)))$$

is not a tautology.

8. Prove that

$$\forall x \bullet \forall y \bullet A \vdash_{ph} \forall y \bullet \forall x \bullet A$$

Deliverable
Your solutions must be typed and submitted by **8:30am on Thursday January 13** in class.