

CS745/ECE725 Fall 2013

Homework 3 (Theorem Proving)

Recall that we showed that graph connectivity is not definable in first-order logic. Such a property is definable in second-order logic as follows:

$$\varphi = \exists P. \forall x. \forall y. \forall z. (C_1 \wedge C_2 \wedge C_3 \wedge C_4)$$

where

$$\begin{aligned} C_1 &\equiv P(x, x) \\ C_2 &\equiv P(x, y) \wedge P(y, z) \Rightarrow P(x, z) \\ C_3 &\equiv P(u, v) \Rightarrow \perp \\ C_4 &\equiv R(x, y) \Rightarrow P(x, y) \end{aligned}$$

First, analyze the above formula to figure out what predicates P and R are. Subsequently, notice that φ intends to capture the fact that v is not reachable from u .

Using PVS, formally prove that φ holds in *any* directed graph iff there does not exist a finite path from vertex u to vertex v in that graph.

Deliverables

You are expected to form teams of 2 people and submit a .pvs and a .prf file through email by **4:00pm on Tuesday, October 15**. Your PVS specification must be fully commented and type checked (no TCCs).