CS745/ECE725 Fall 2013
Homework 1

(Propositional Logic and SAT Solving)

1. A prisoner has two guards. One is always honest and the other always lies. The prisoner does not know which one is honest and which one is the liar. There are two gates in the prison: one goes to freedom, another goes to sudden and even immediate death! Both guards know which gate goes to where. How can the prisoner ensure freedom by asking one and only one question from one guard? Support your answer using propositional logic.

2. Let $p \overline{\vee} q$ denote the connective with the following semantics:

p	q	$p \overline{\vee} q$
T	**T**	**F**
T	**F**	**F**
F	**T**	**F**
F	**F**	**T**

   Show that all binary propositional connectives (i.e., $\wedge, \vee, \Rightarrow, \Leftrightarrow$) can be defined using $\overline{\vee}$.

3. Show that connectives $\wedge, \vee, \Rightarrow$ cannot be defined using connectives $\neg$ and $\Leftrightarrow$ only.

4. Show that $\hookrightarrow (A \Rightarrow (B \Rightarrow C)) \Leftrightarrow ((A \Rightarrow B) \Rightarrow (A \Rightarrow C))$ in sequent calculus.

5. Let S be a set of formulas. An *associated sequent* for S is a sequent $\Gamma \hookrightarrow \neg\Delta$, where Γ, Δ is a partition of S (i.e., $\Gamma \cap \Delta = \emptyset$ and $\Gamma \cup \Delta = S$), and $\neg\Delta$ denotes the set $\{\neg X \mid X \in \Delta\}$. Show that if an associated sequent for S has a proof, then every associated sequent does.

6. Consider the following constraints:

   - If the weather is cold and the patient has been exposed to influenza virus, then the patient will get in influenza.
   - If the patient has influenza, then the patient will have runny nose.
   - If the patient has influenza, then the patient will have a temperature.
   - If the patient is a newborn and has been exposed to the measles virus, then the patient will get measles.
   - If the patient has measles, then the patient will have a temperature.
   - If the patient has measles then the patient will have spots.
   - If the patient has a temperature and the weather is cold, then the patient should have a hot rum drink.

   Use the SMT-solver **Yices** to determine

   > *"if a patient who has been exposed to influenza virus while the weather is cold should drink hot rum drink."*

   Also, demonstrate your answer obtained by Yices by manually applying one of the SAT-solving algorithms.

Deliverable
Your solutions must be typed and submitted by **4pm on Tuesday September 17** in class.