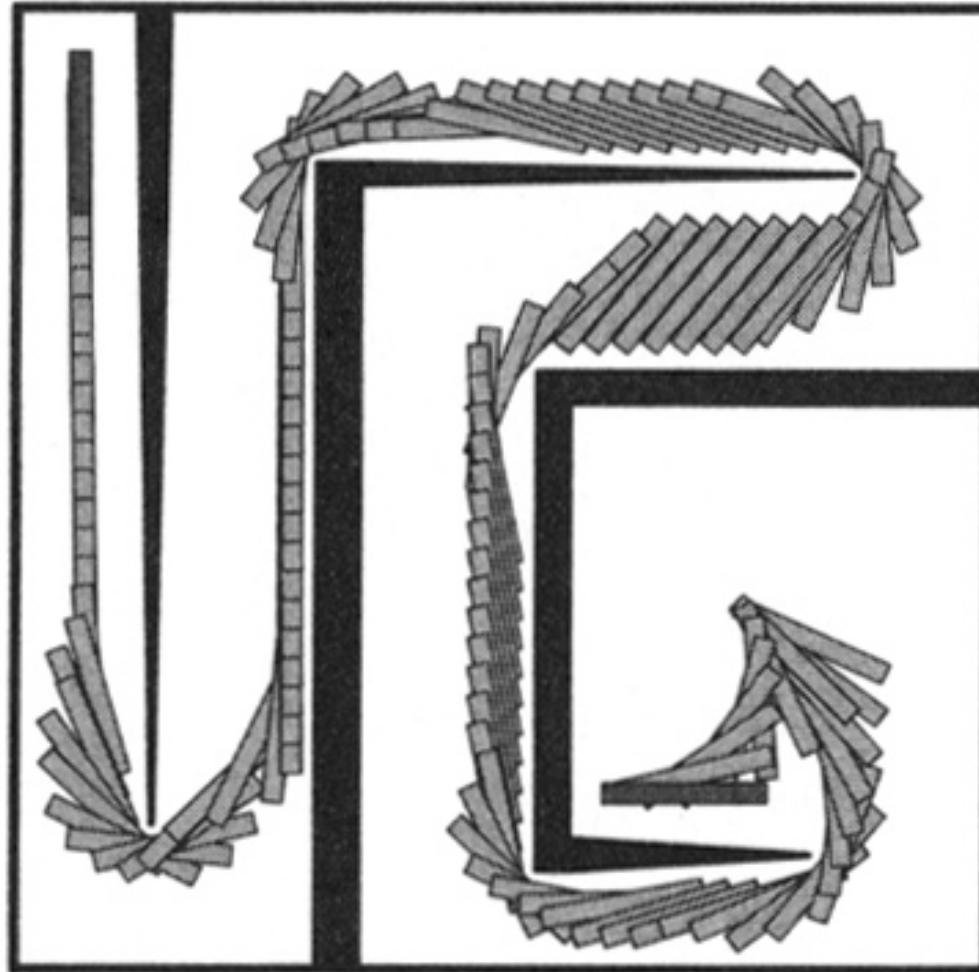


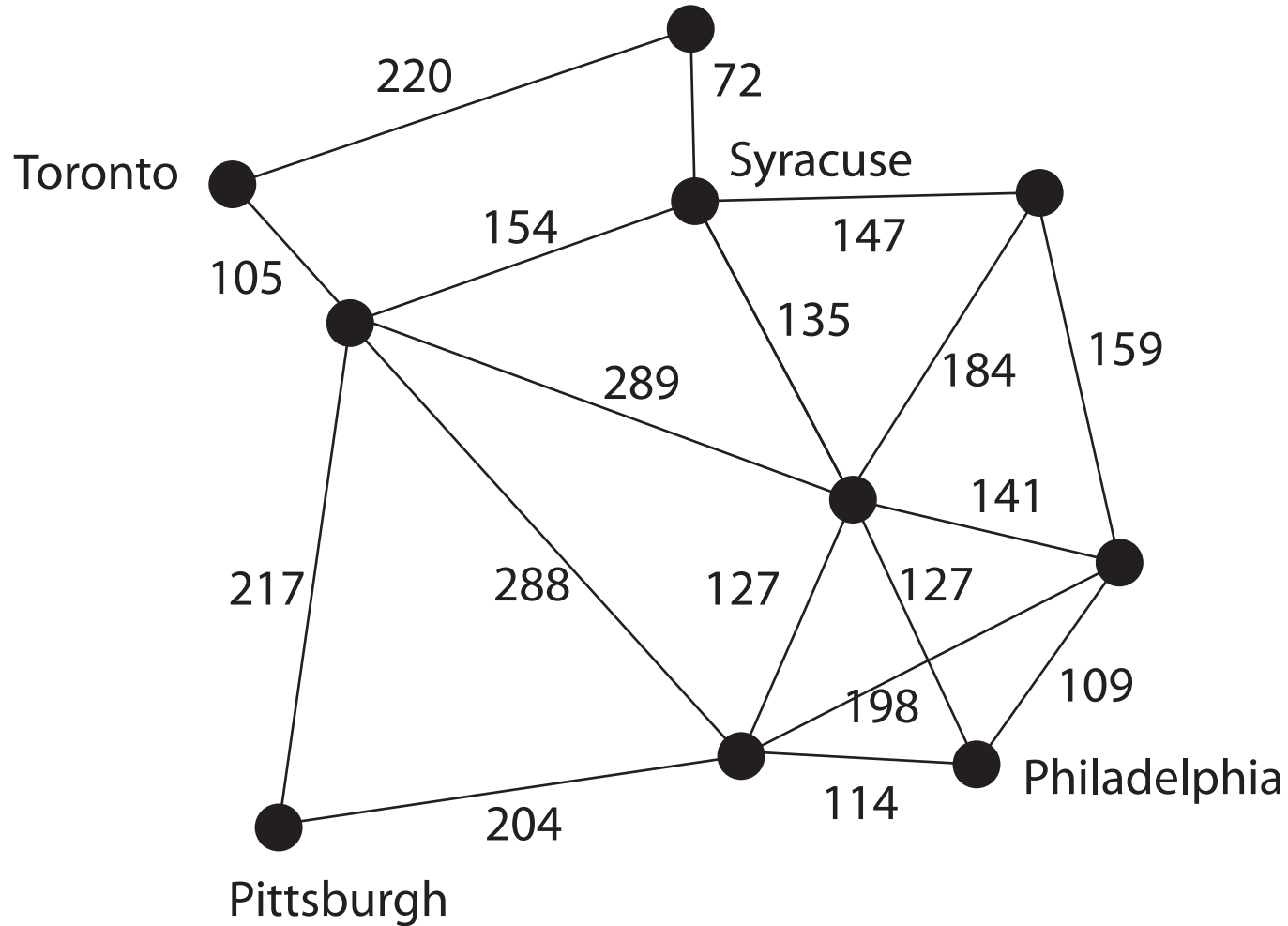
Shortest Path Problem

Motion Planning



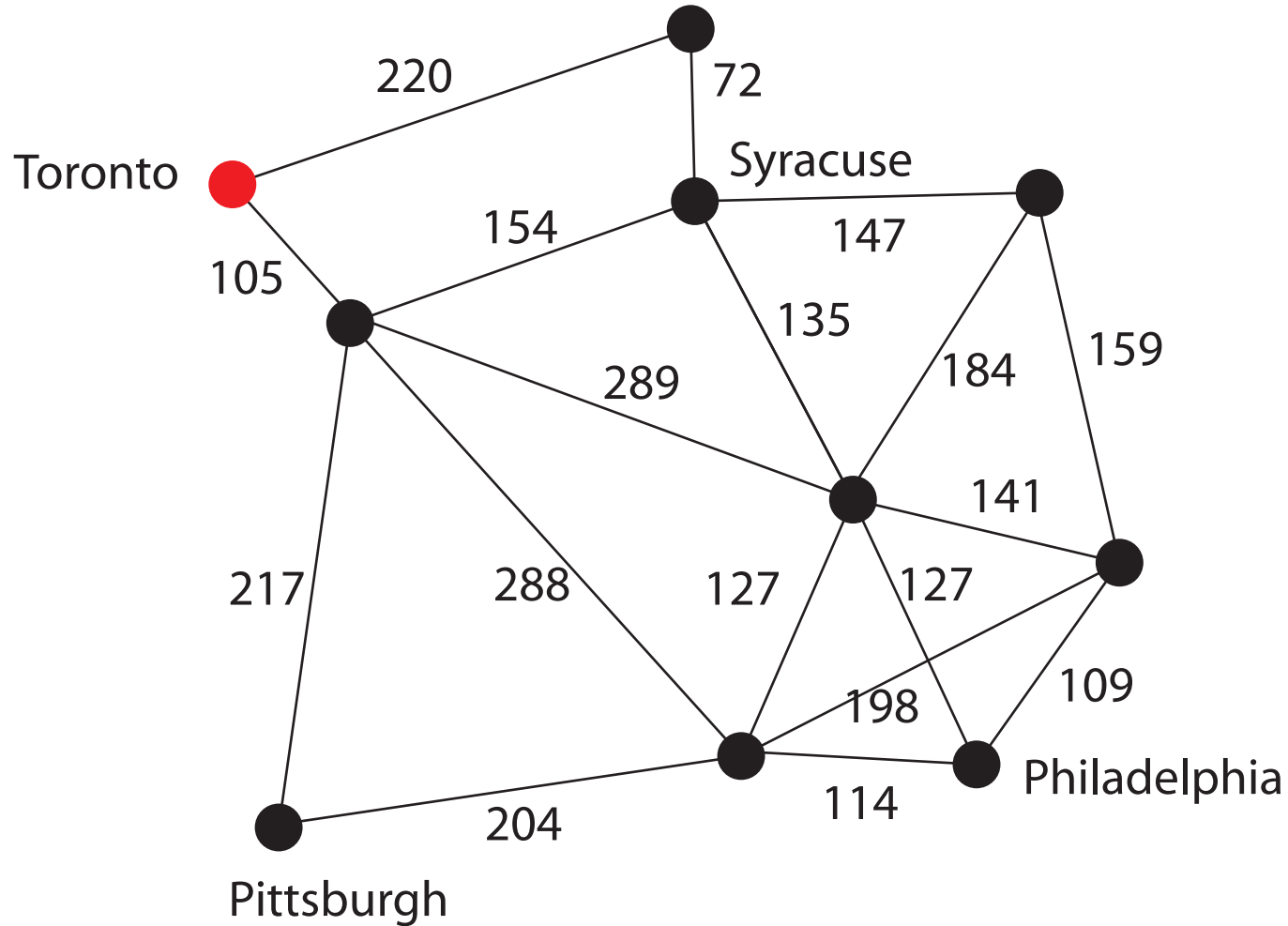
Shortest Path in a Graph

Dijkstra's Algorithm, 1959



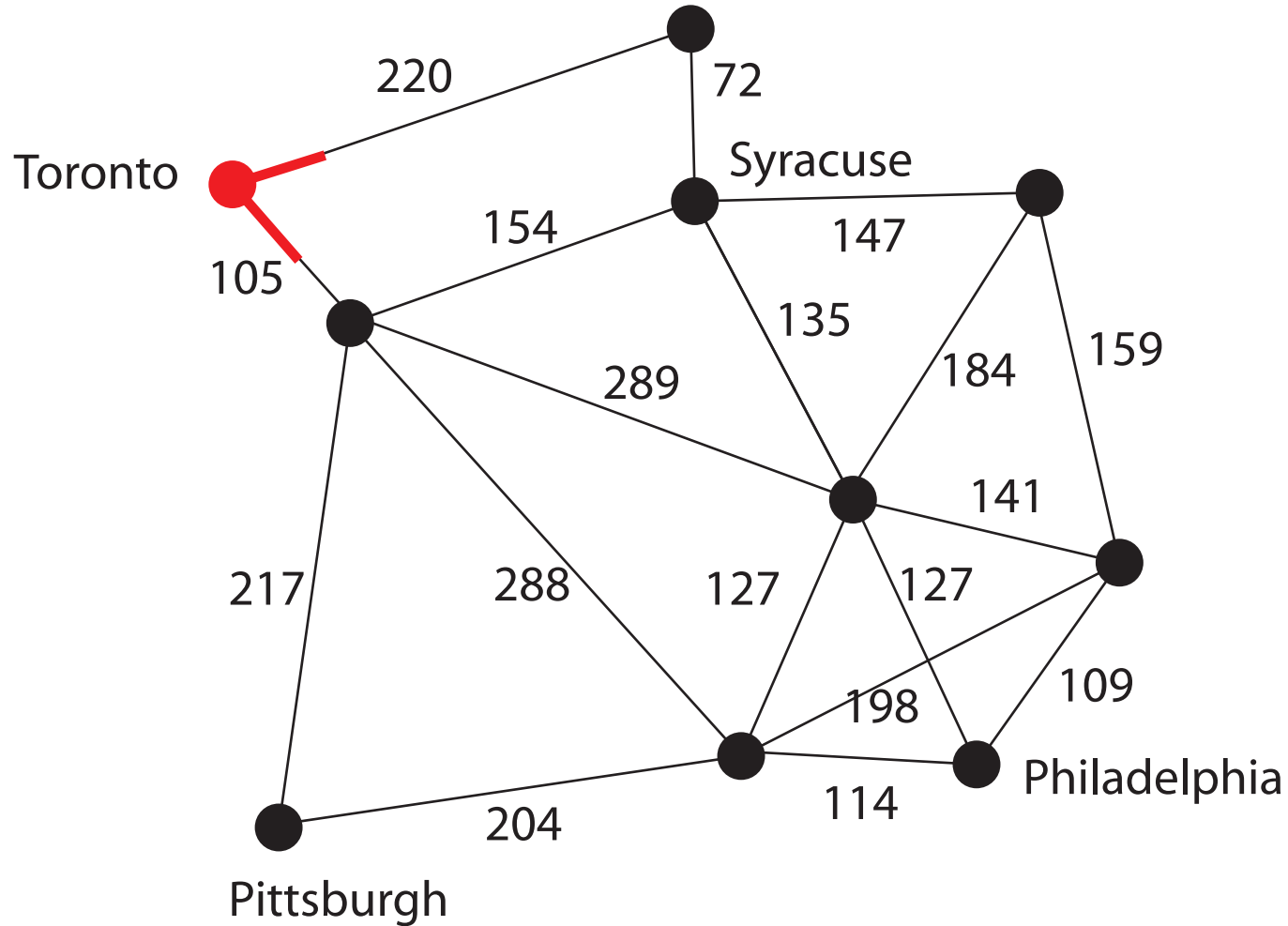
Shortest Path in a Graph

Dijkstra's Algorithm, 1959



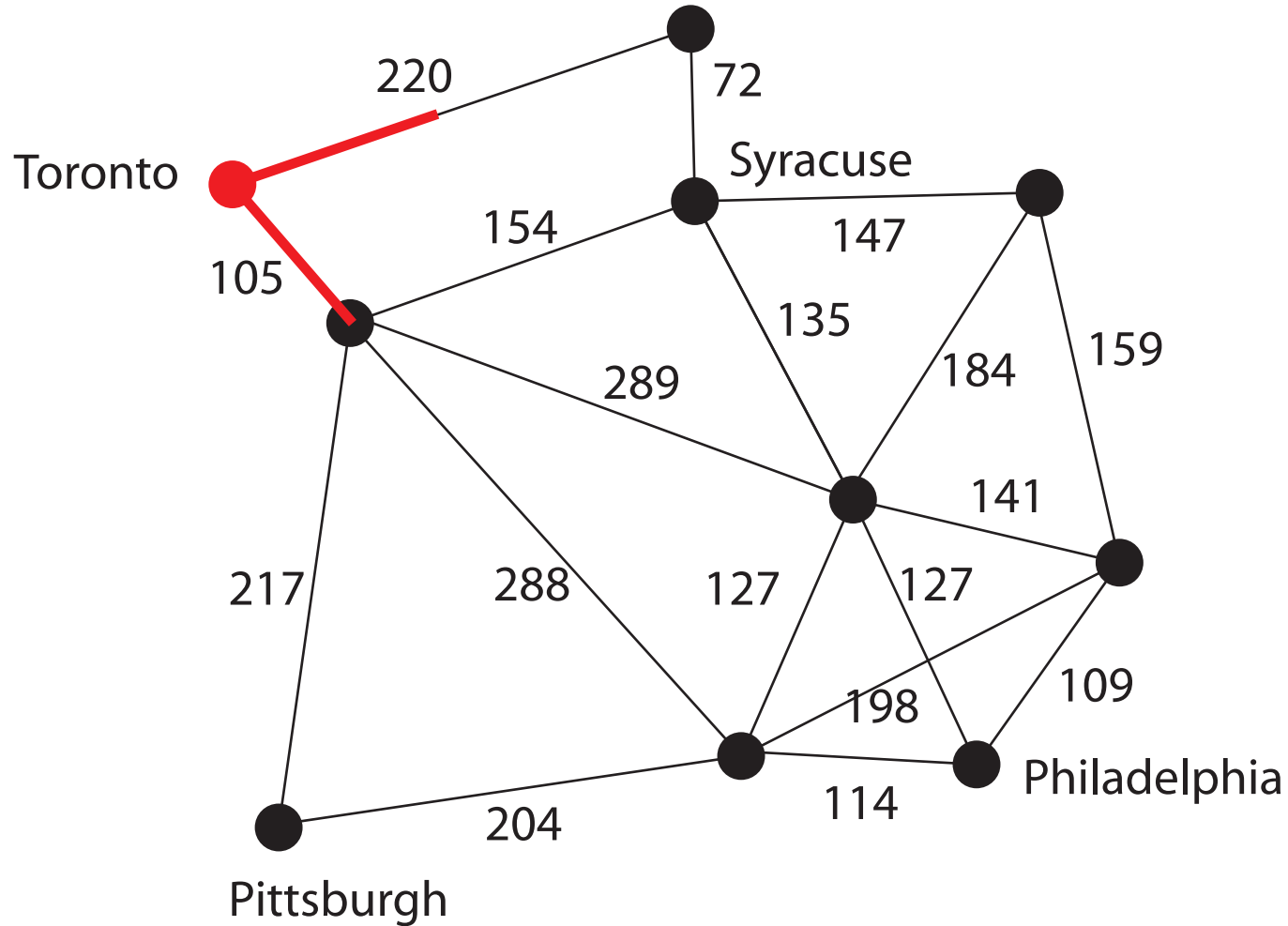
Shortest Path in a Graph

Dijkstra's Algorithm, 1959



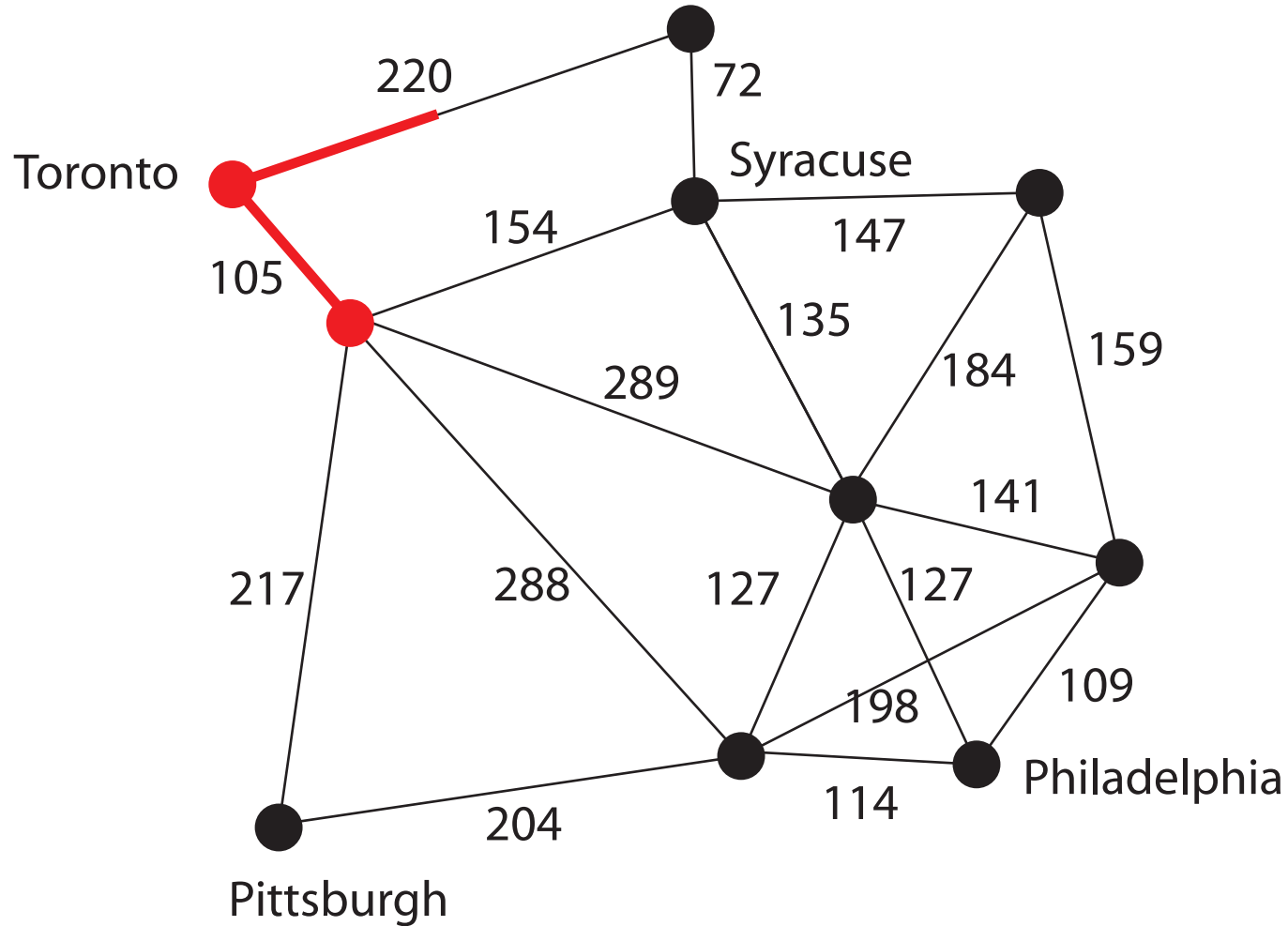
Shortest Path in a Graph

Dijkstra's Algorithm, 1959



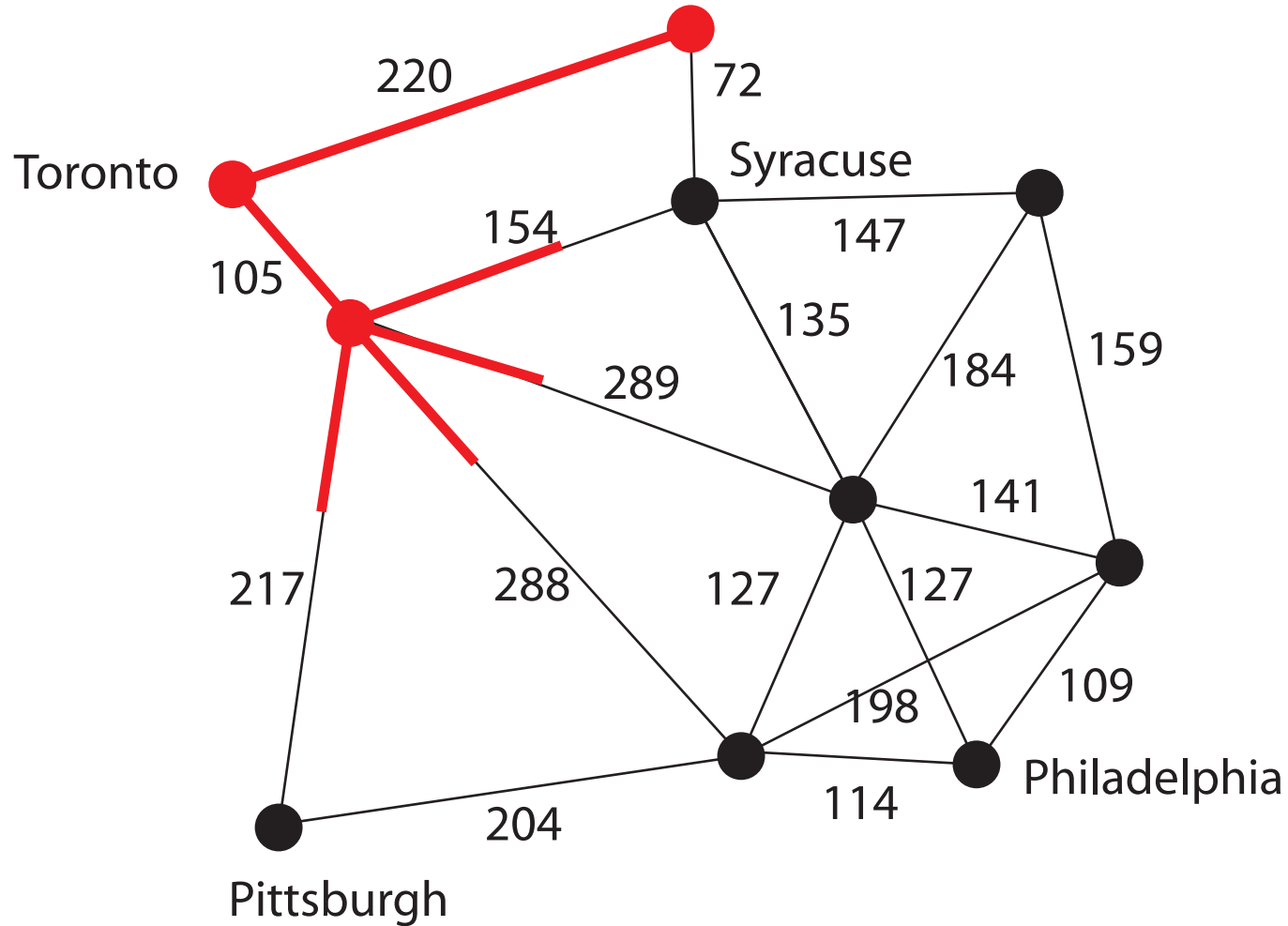
Shortest Path in a Graph

Dijkstra's Algorithm, 1959



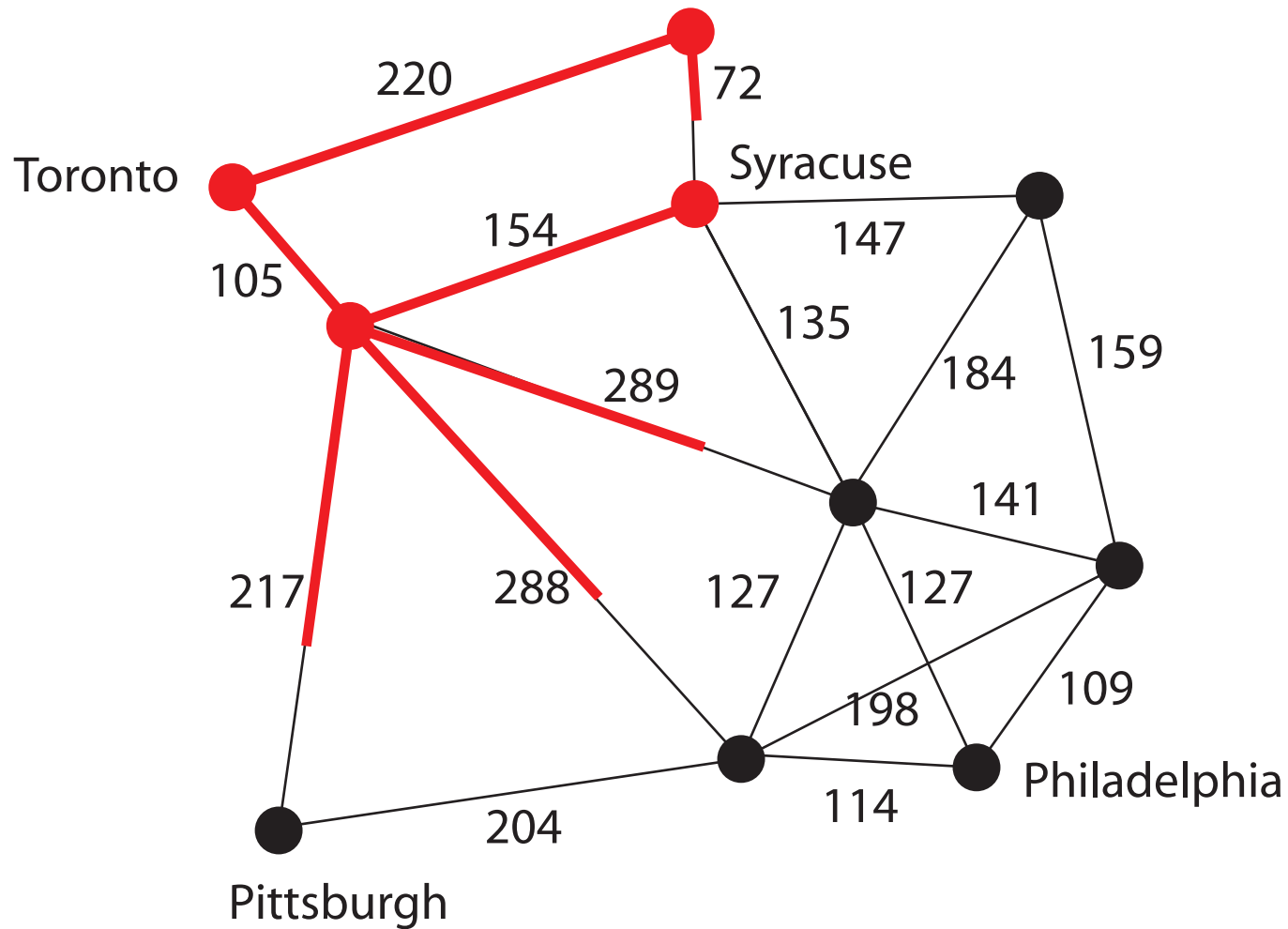
Shortest Path in a Graph

Dijkstra's Algorithm, 1959



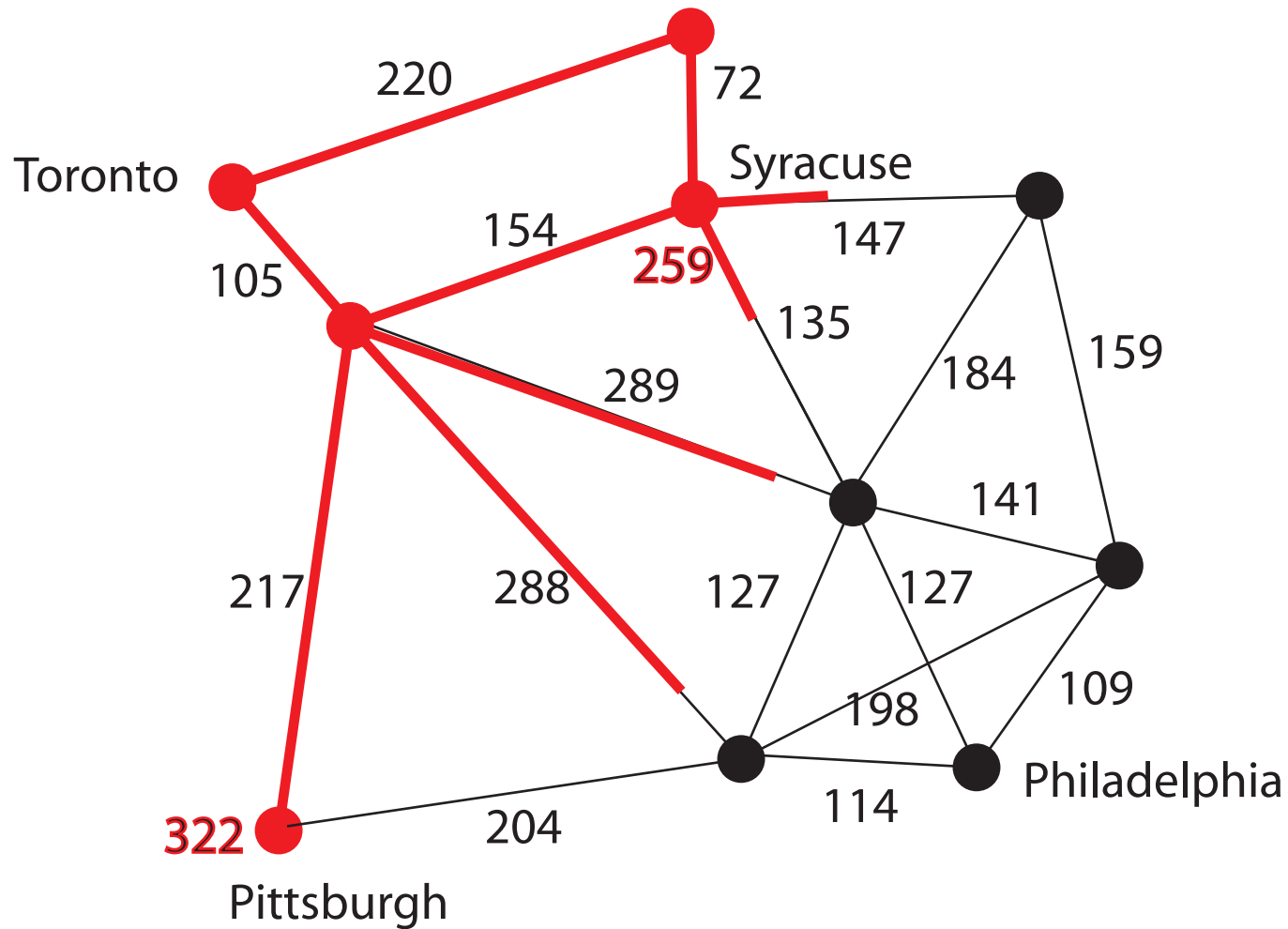
Shortest Path in a Graph

Dijkstra's Algorithm, 1959



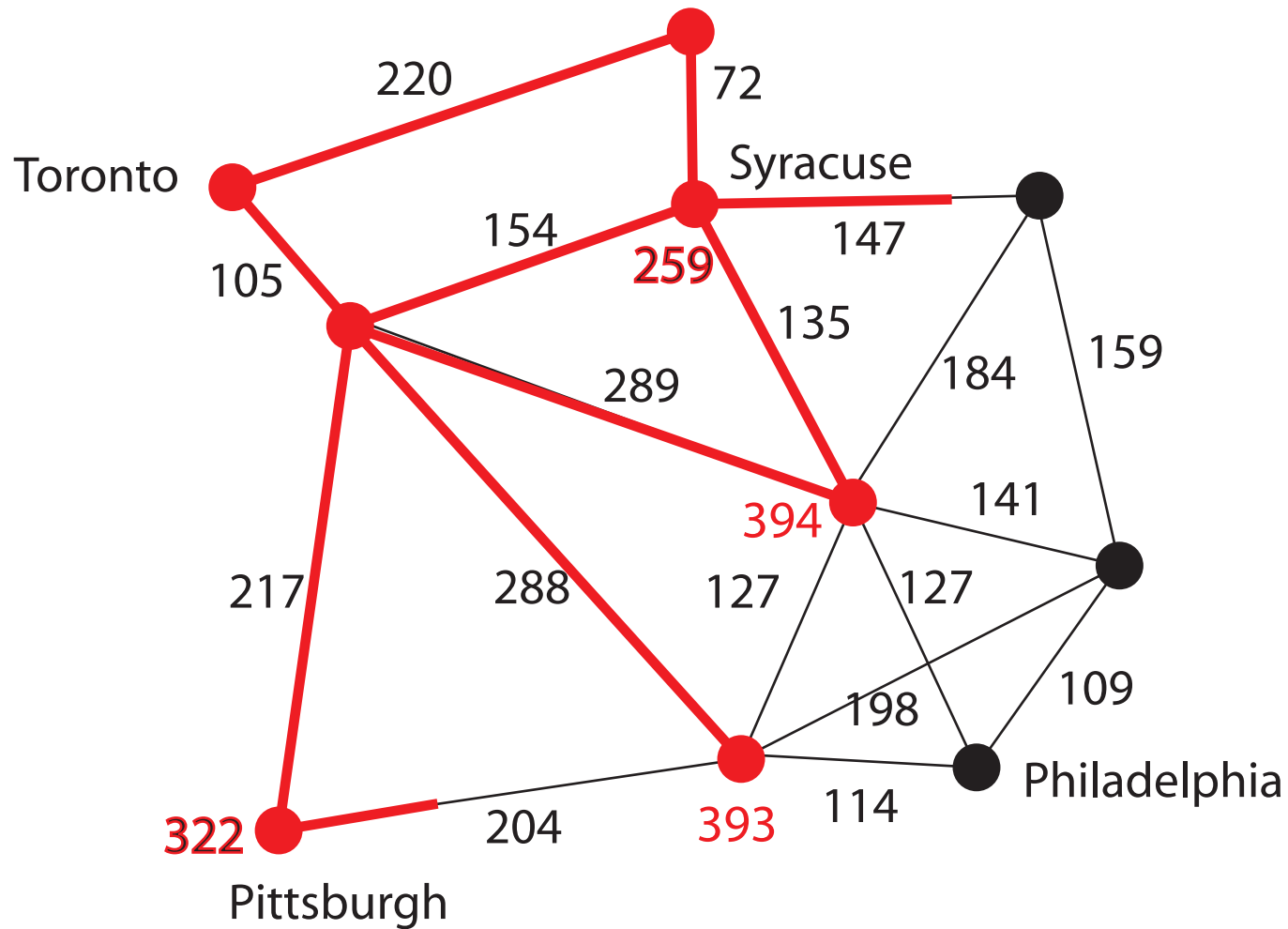
Shortest Path in a Graph

Dijkstra's Algorithm, 1959



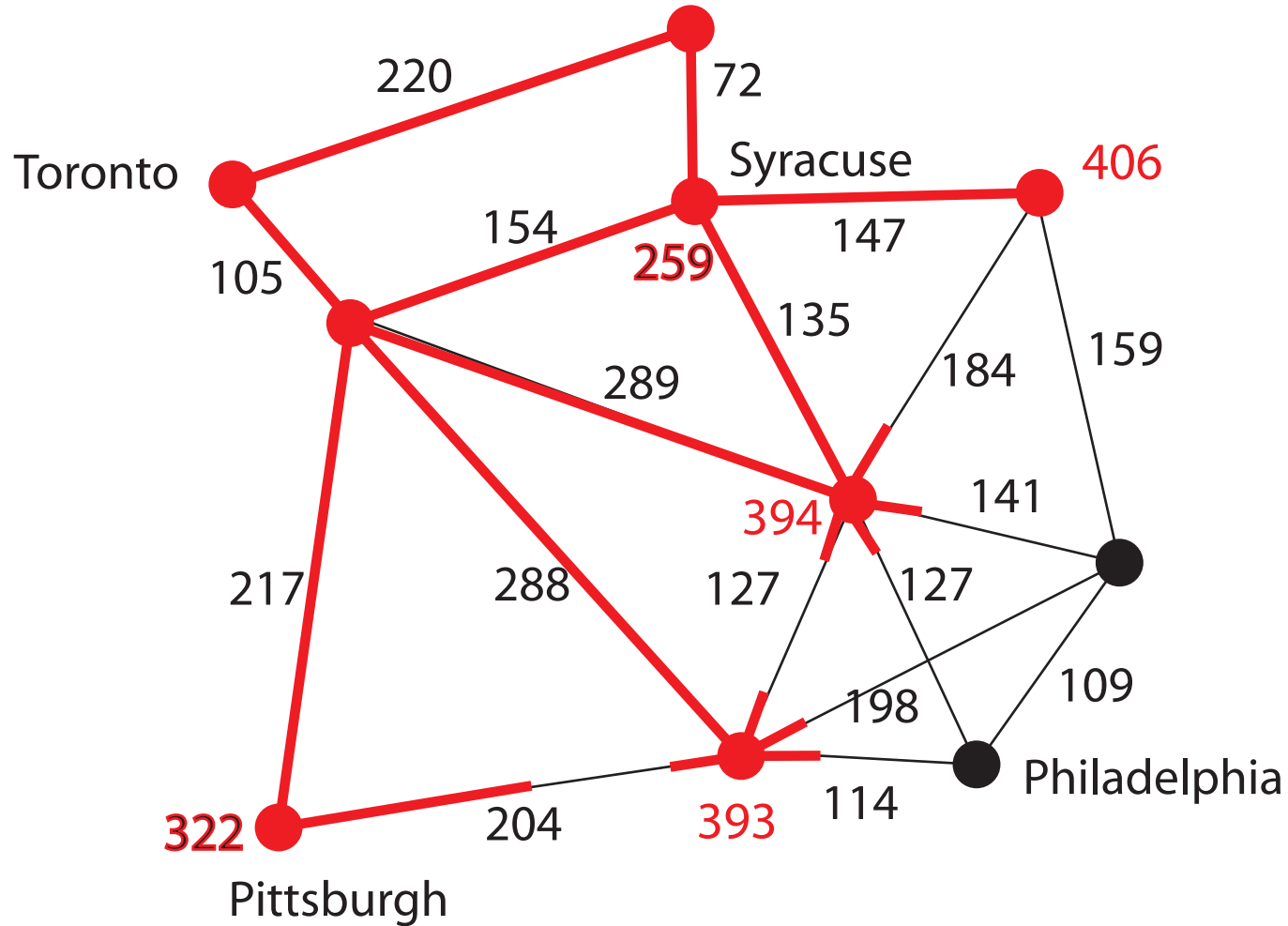
Shortest Path in a Graph

Dijkstra's Algorithm, 1959



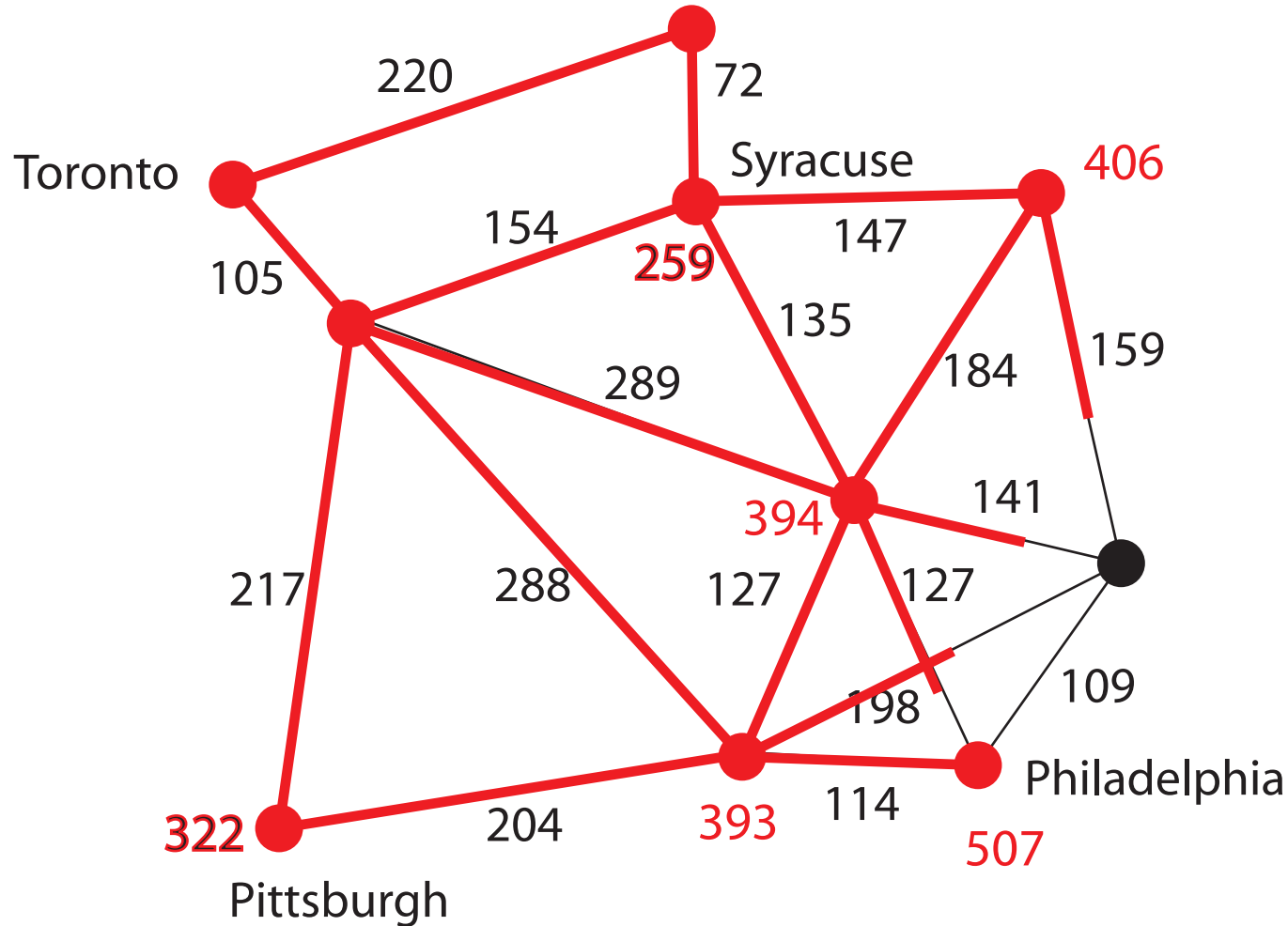
Shortest Path in a Graph

Dijkstra's Algorithm, 1959



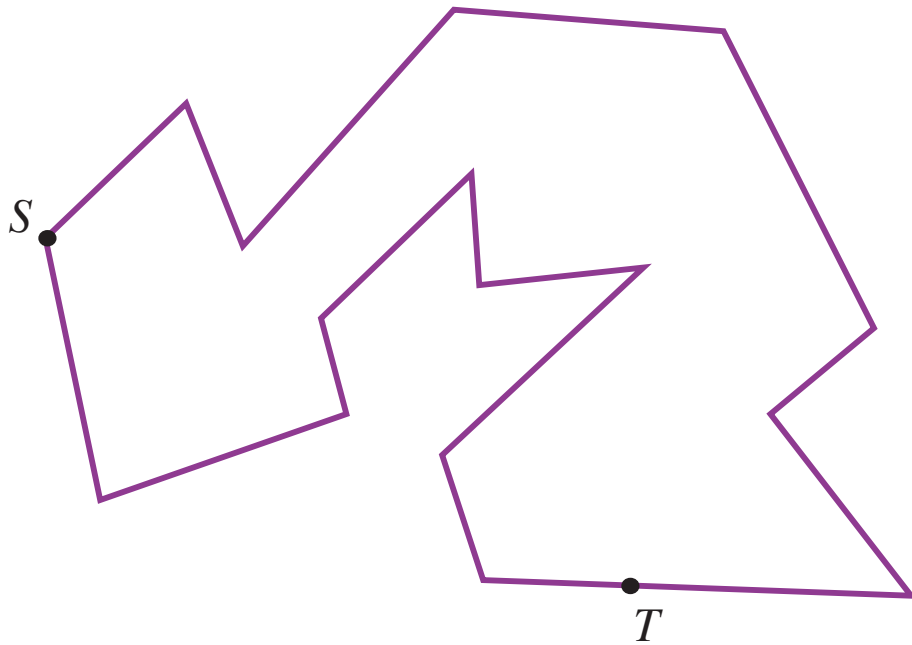
Shortest Path in a Graph

Dijkstra's Algorithm, 1959 -- $O(m + n \log n)$ Fredman & Tarjan, 1987
using Fibonacci heaps

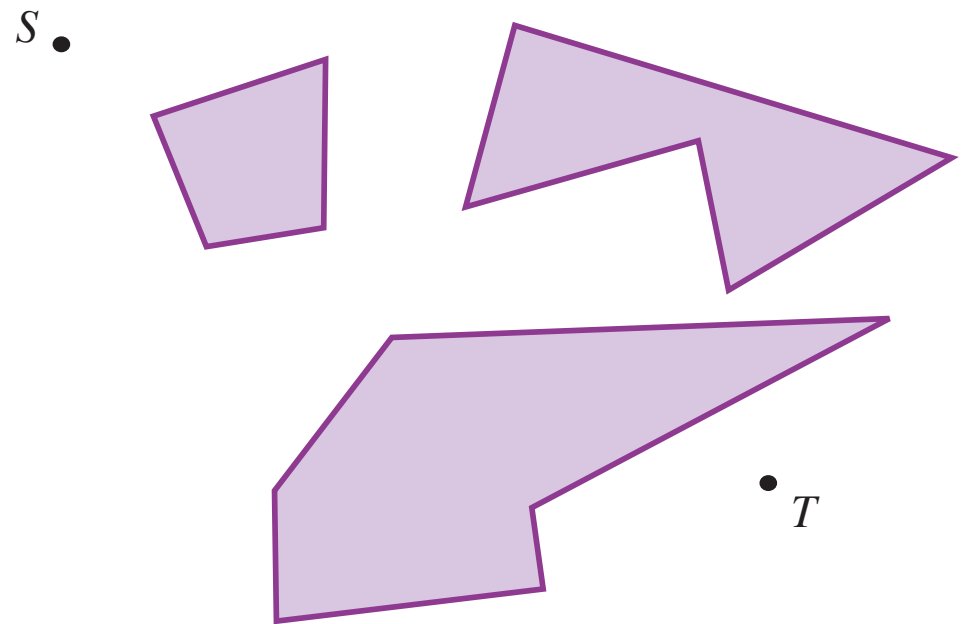


Geometric Shortest Paths

Polygon

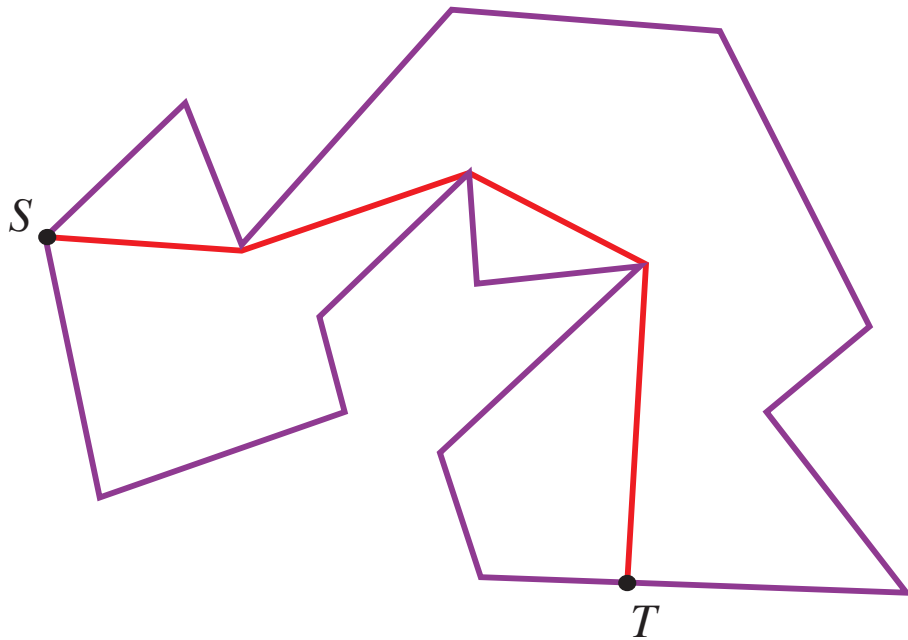


Polygonal Domain

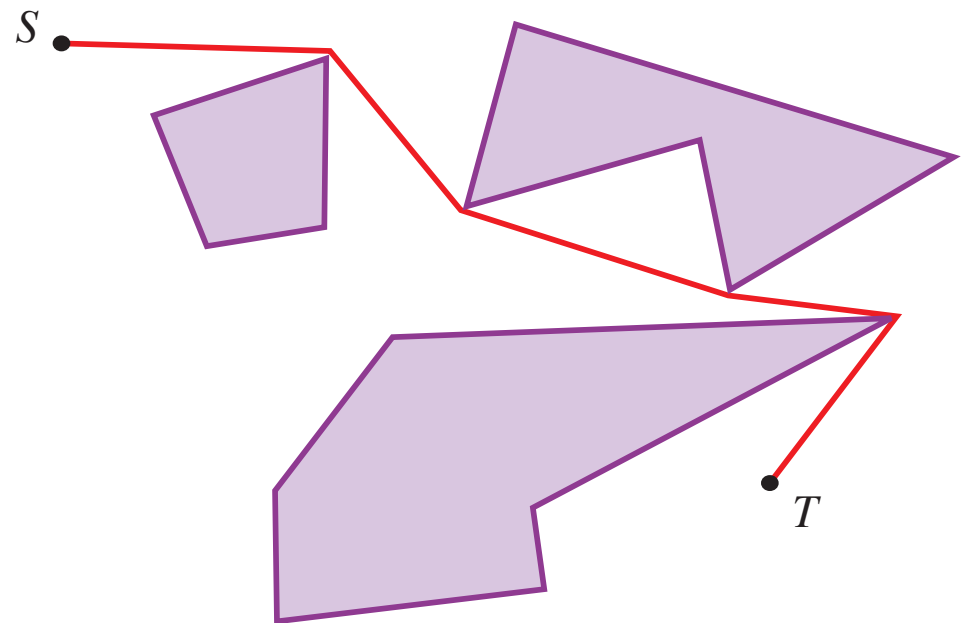


Geometric Shortest Paths

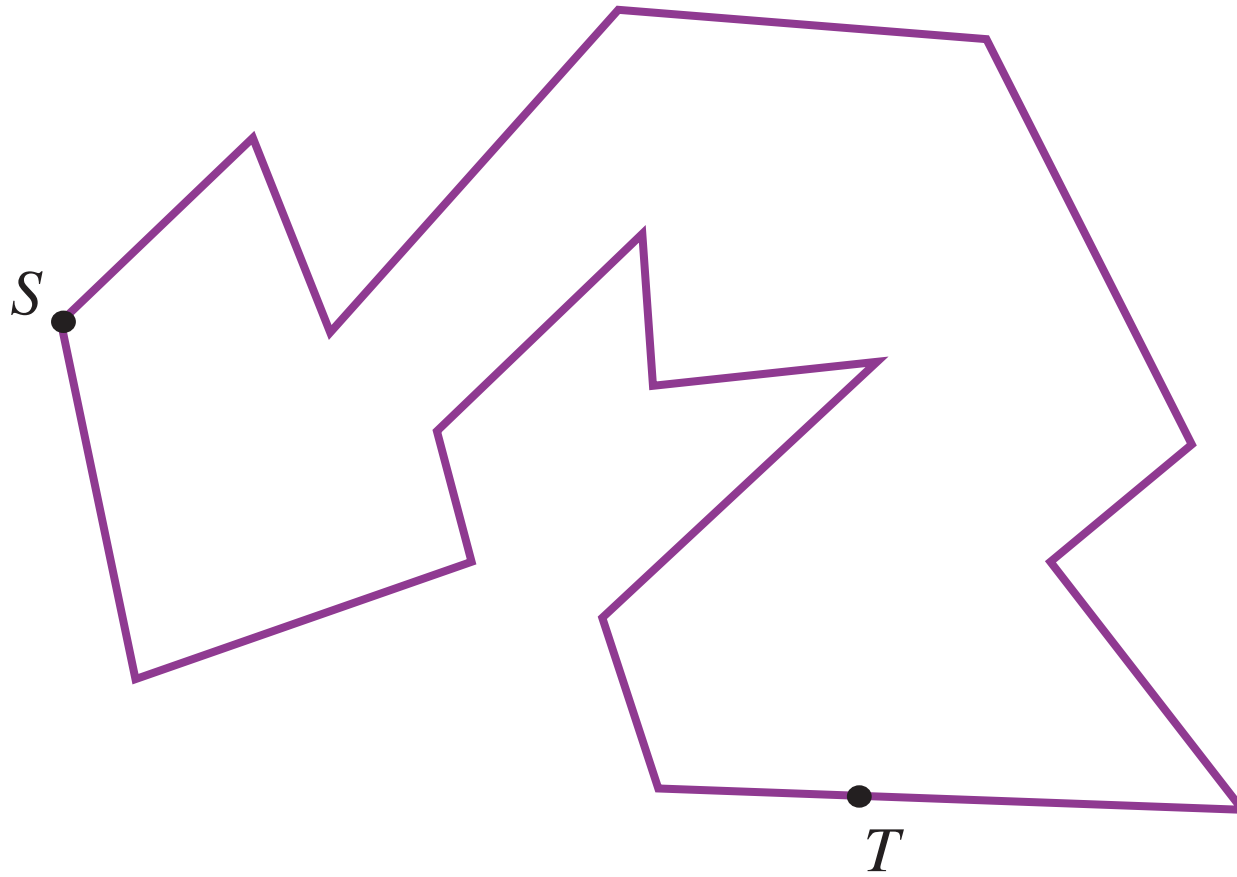
Polygon



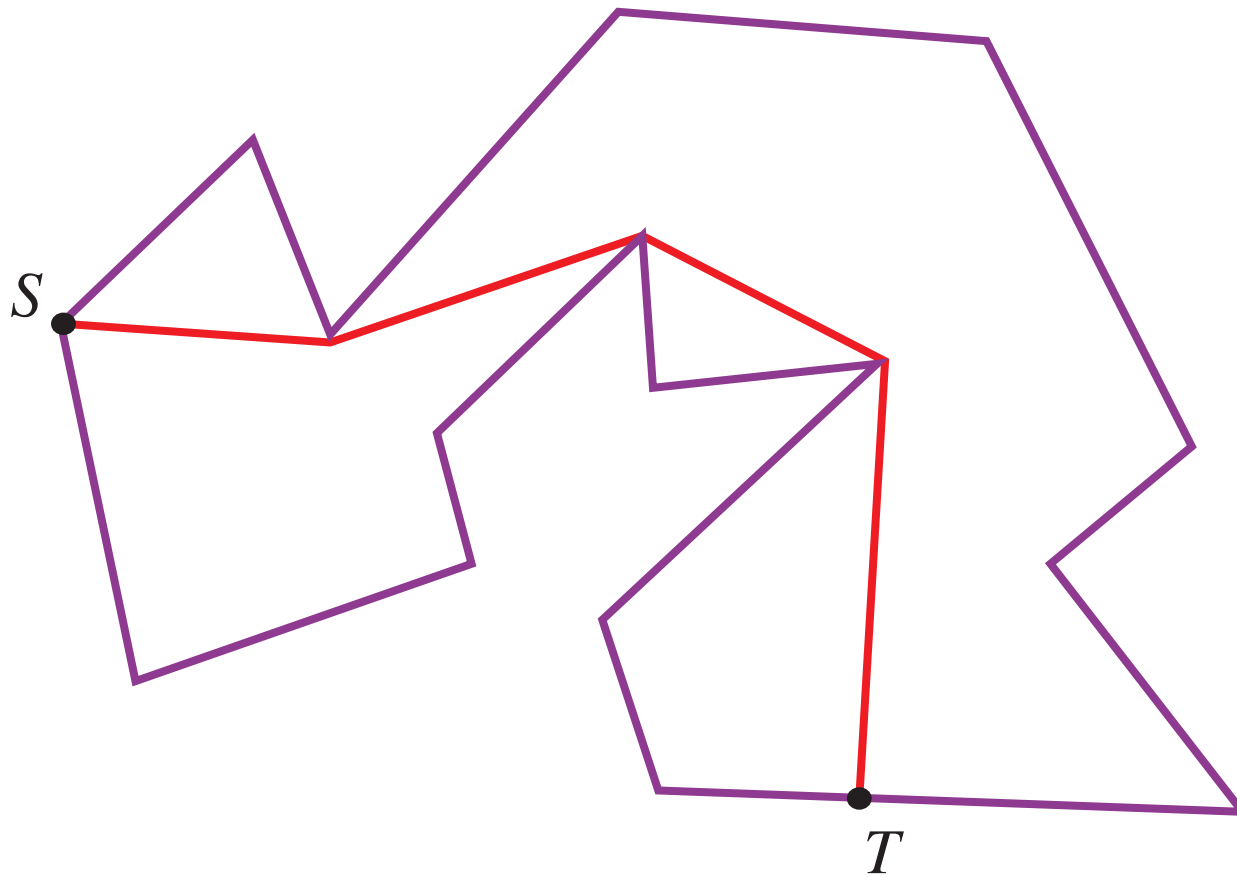
Polygonal Domain



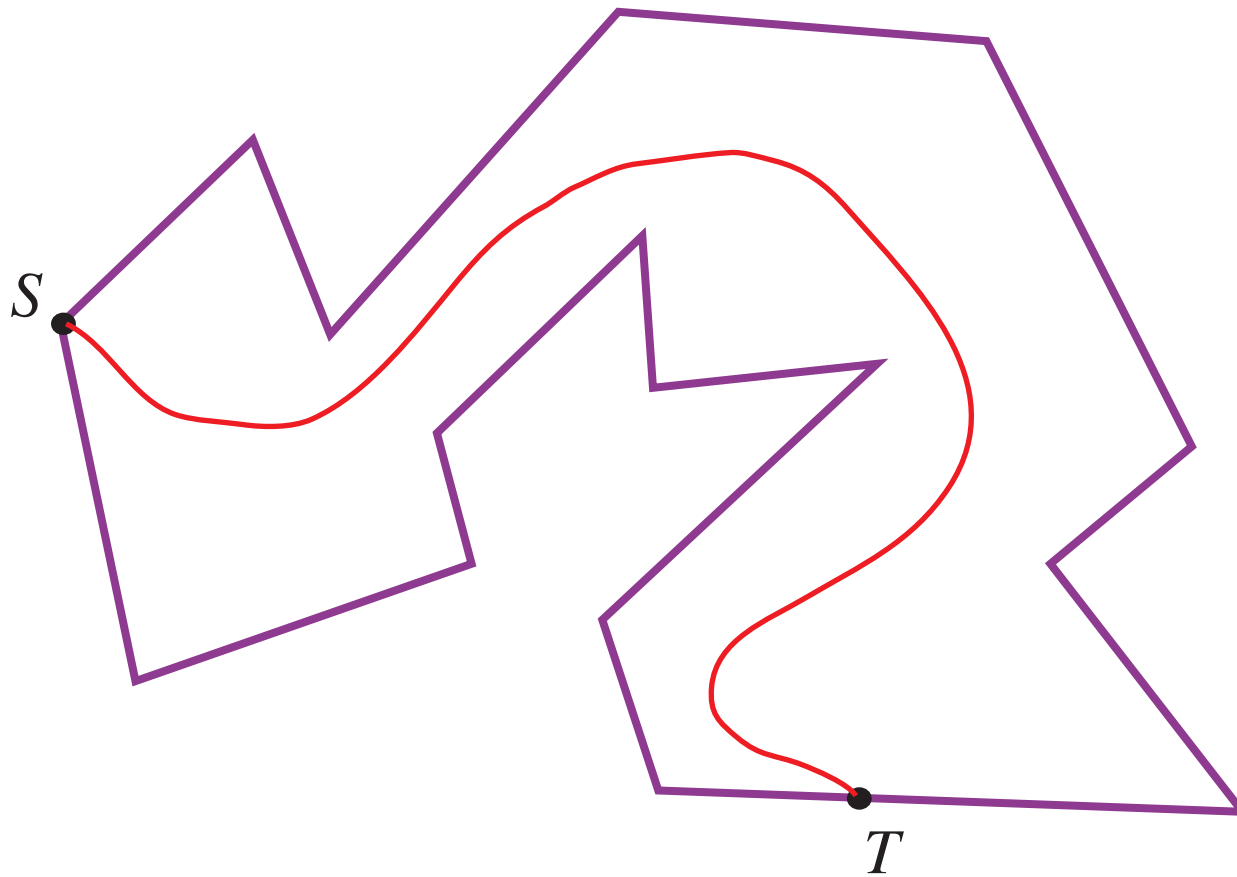
Geometric Shortest Paths -- Polygon



Geometric Shortest Paths -- Polygon

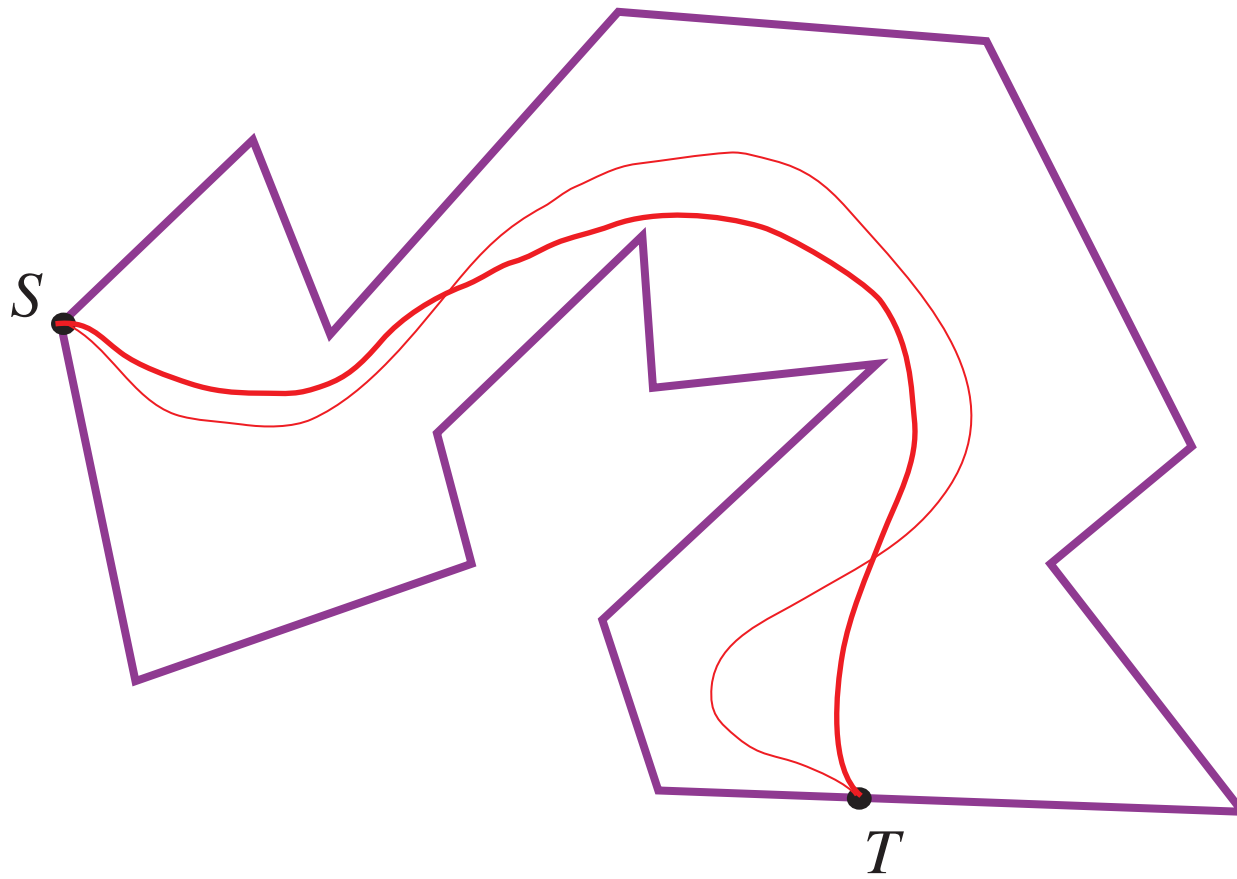


Geometric Shortest Paths -- Polygon



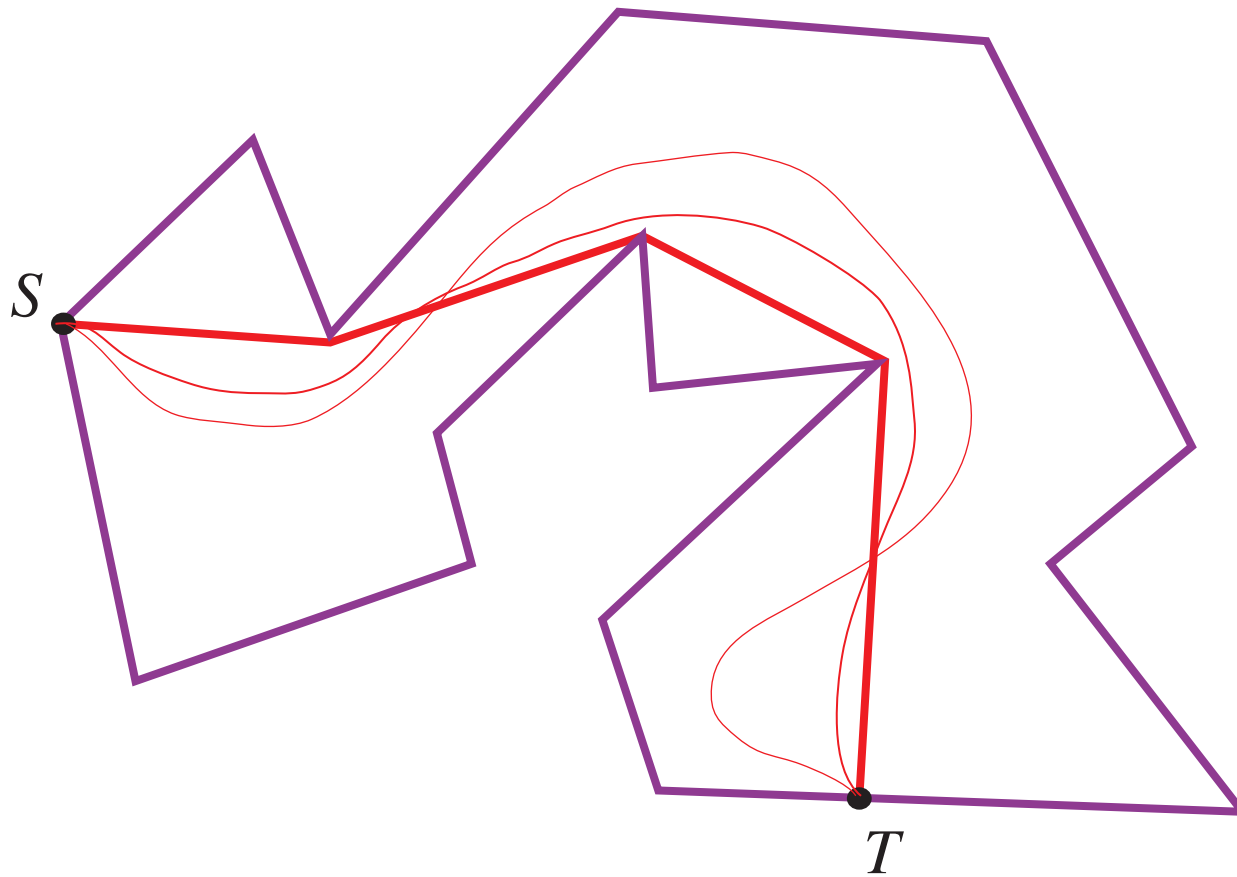
elastic band solution

Geometric Shortest Paths -- Polygon



elastic band solution

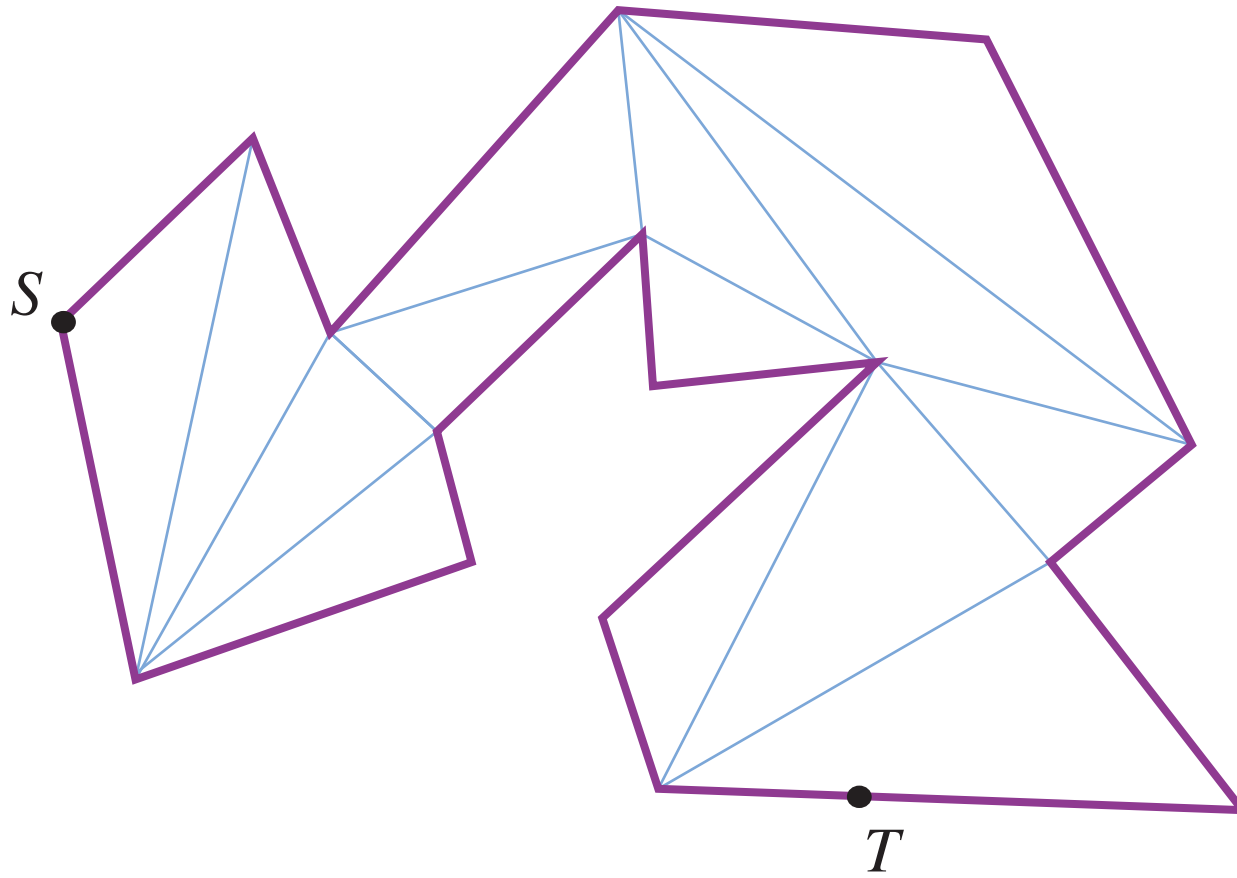
Geometric Shortest Paths -- Polygon



elastic band solution (locally shortest)

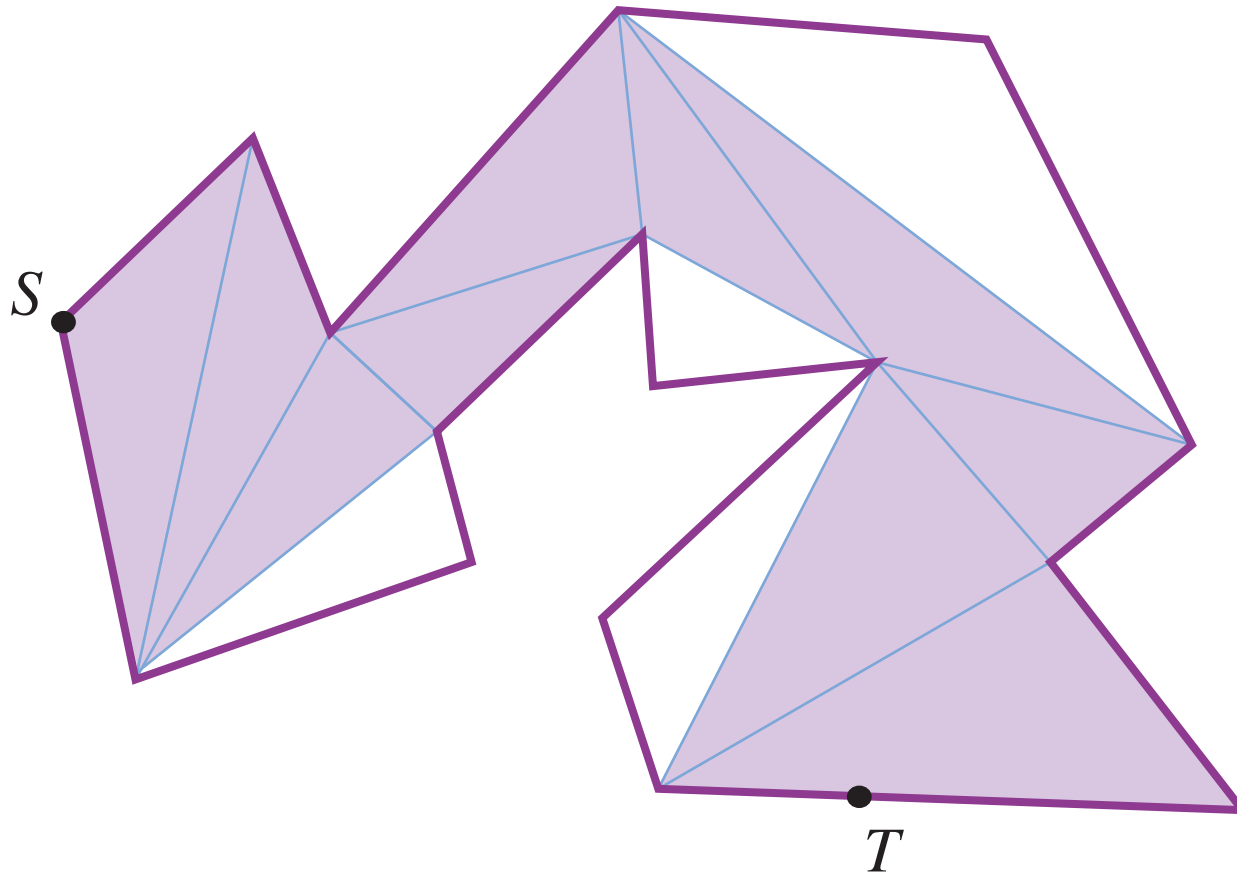
Geometric Shortest Paths -- Polygon

Funnel Algorithm



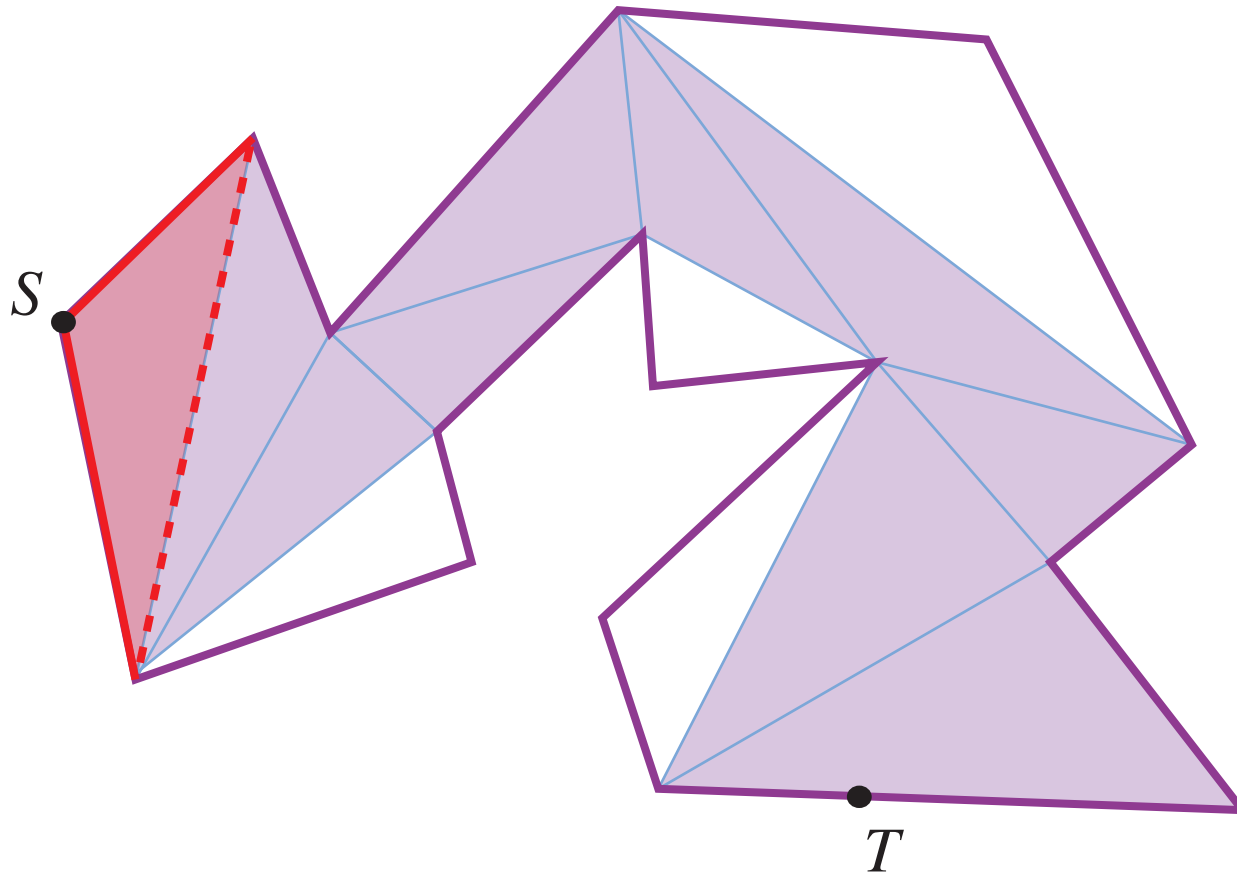
Geometric Shortest Paths -- Polygon

Funnel Algorithm



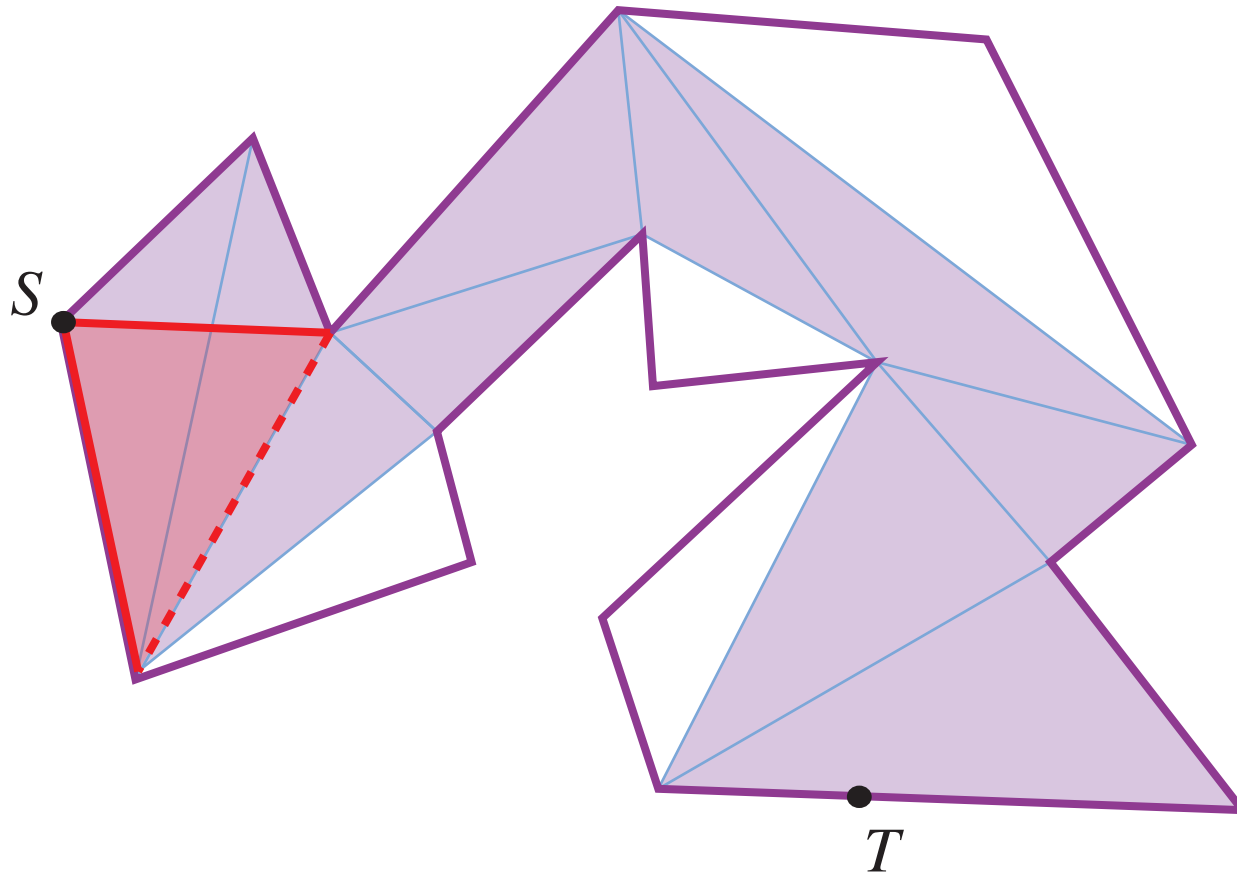
Geometric Shortest Paths -- Polygon

Funnel Algorithm



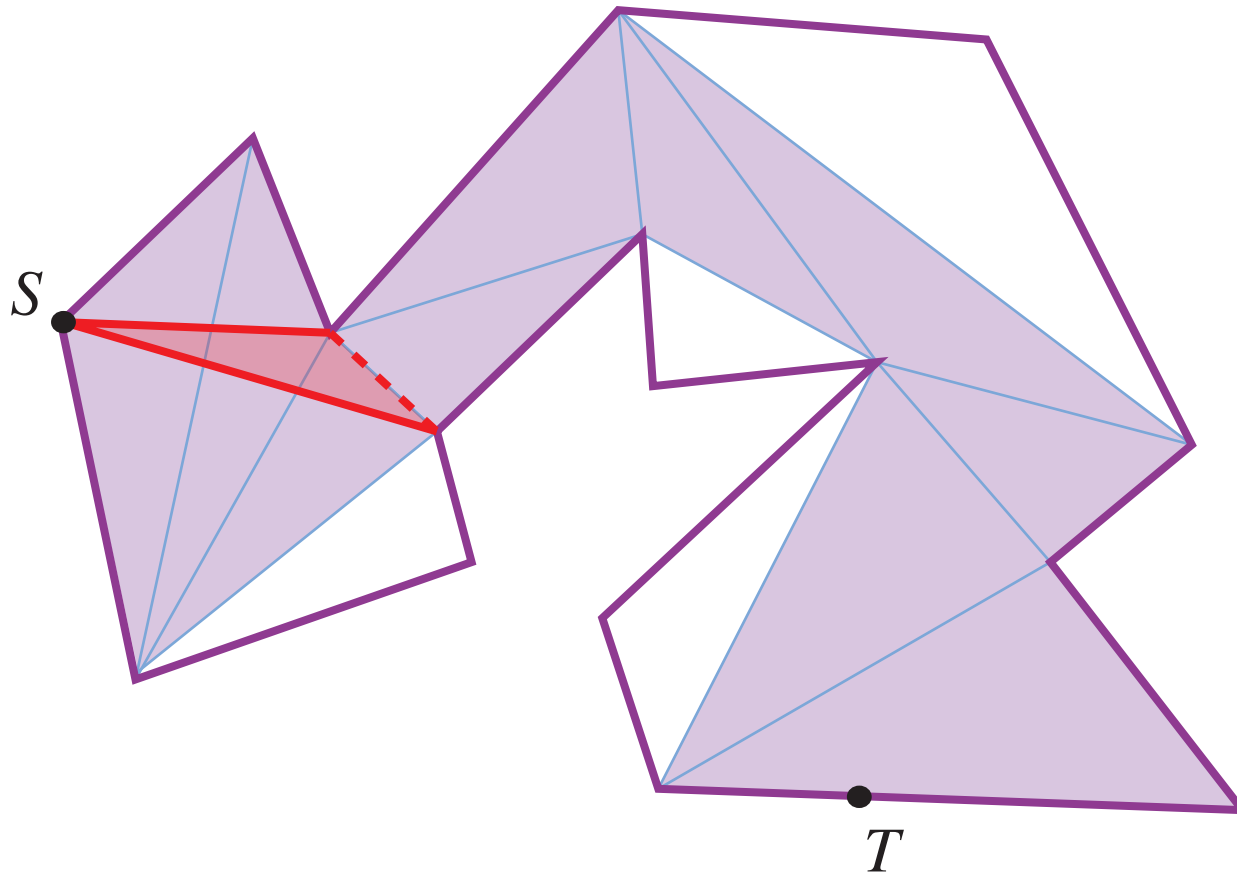
Geometric Shortest Paths -- Polygon

Funnel Algorithm



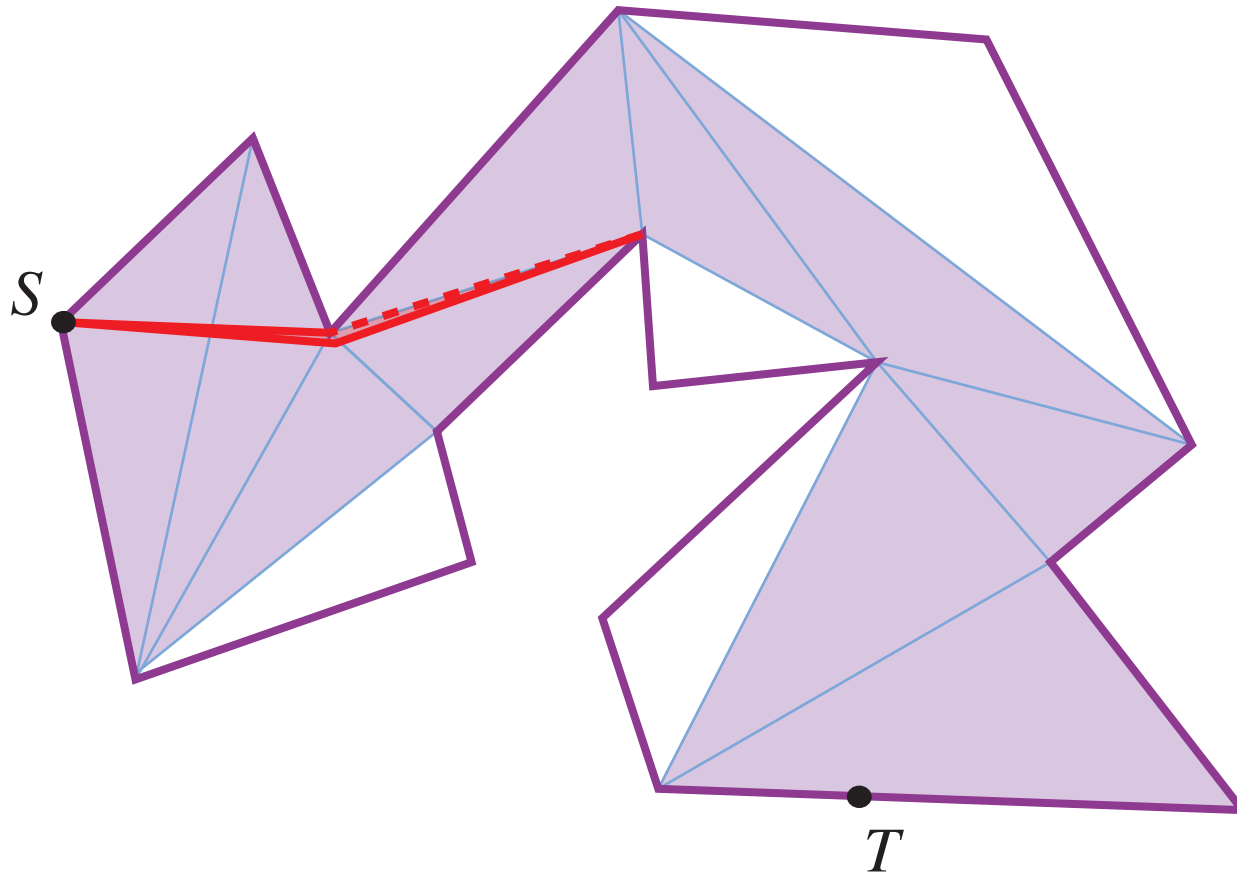
Geometric Shortest Paths -- Polygon

Funnel Algorithm



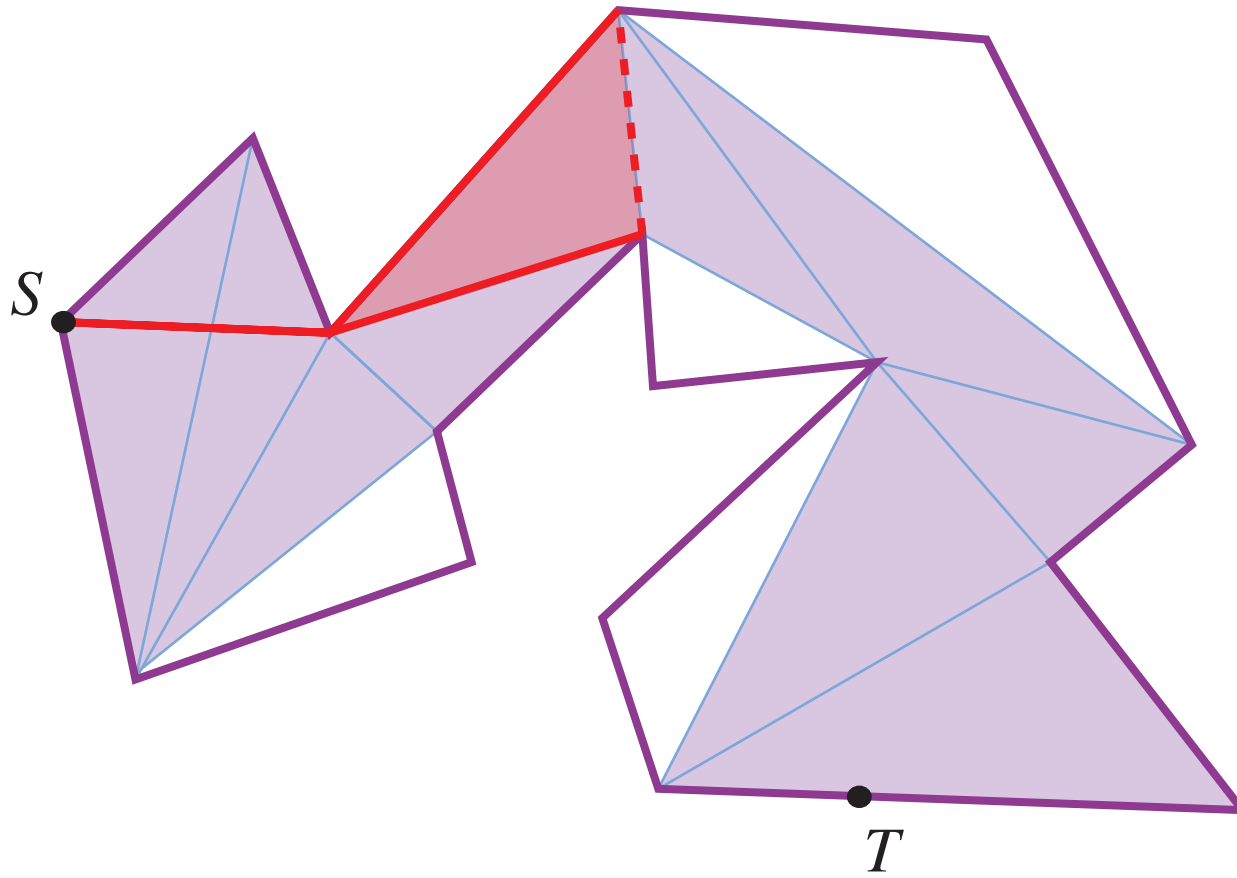
Geometric Shortest Paths -- Polygon

Funnel Algorithm



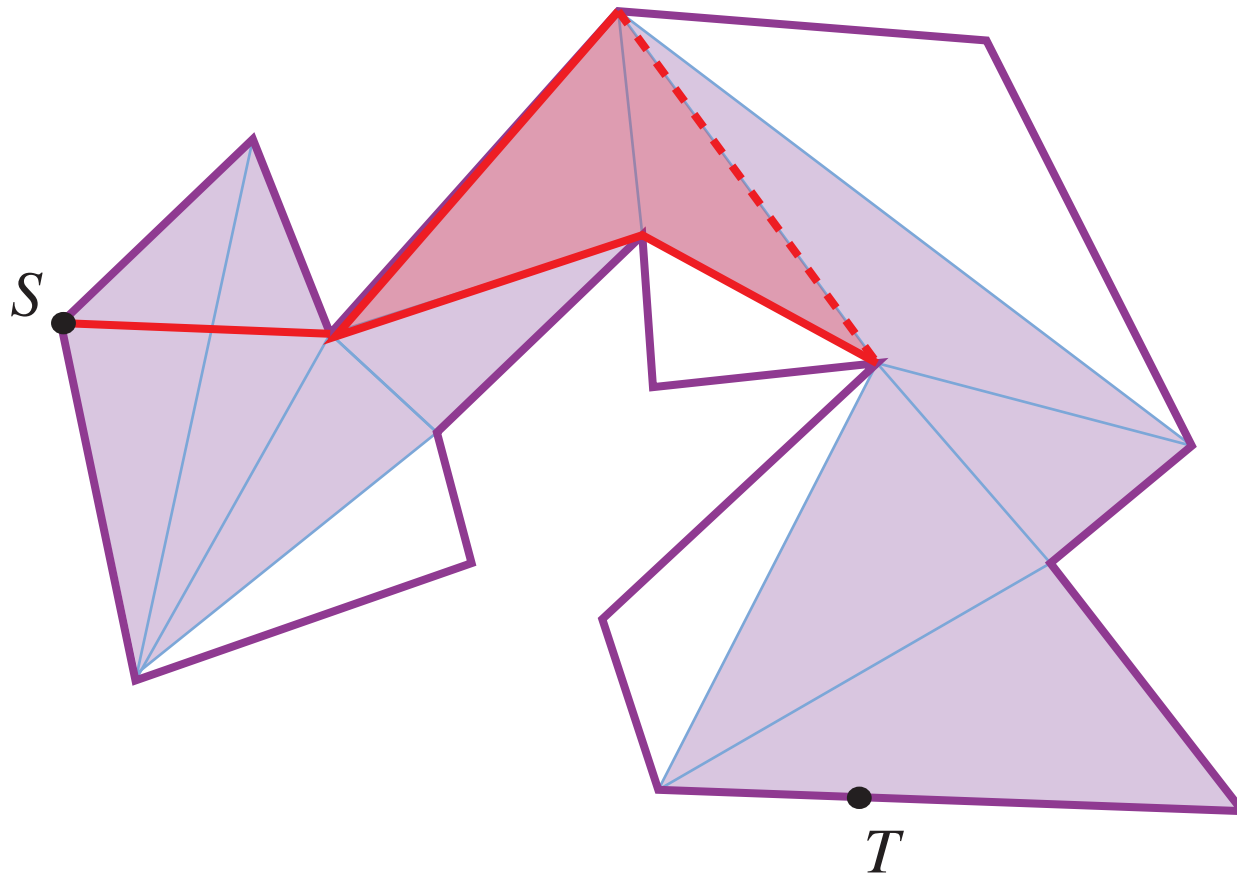
Geometric Shortest Paths -- Polygon

Funnel Algorithm



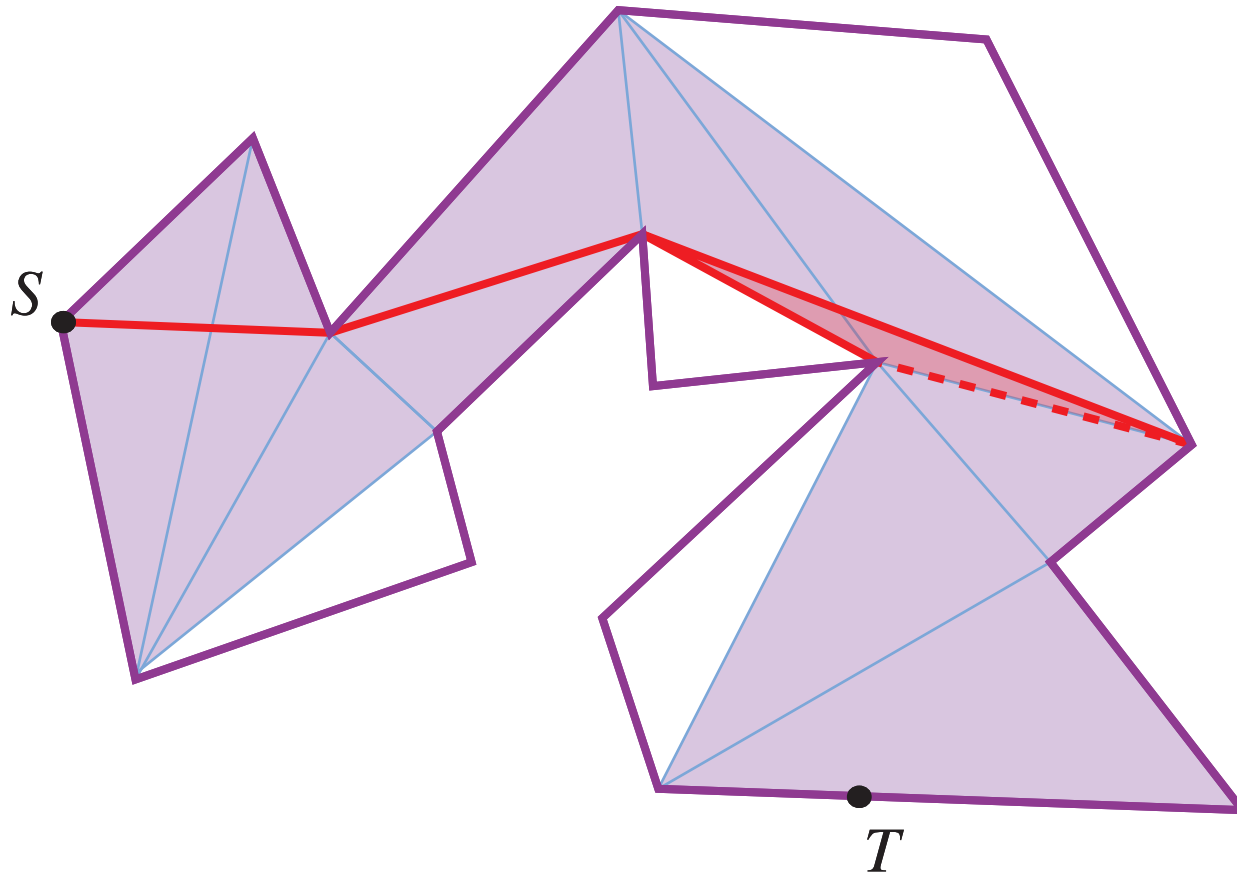
Geometric Shortest Paths -- Polygon

Funnel Algorithm



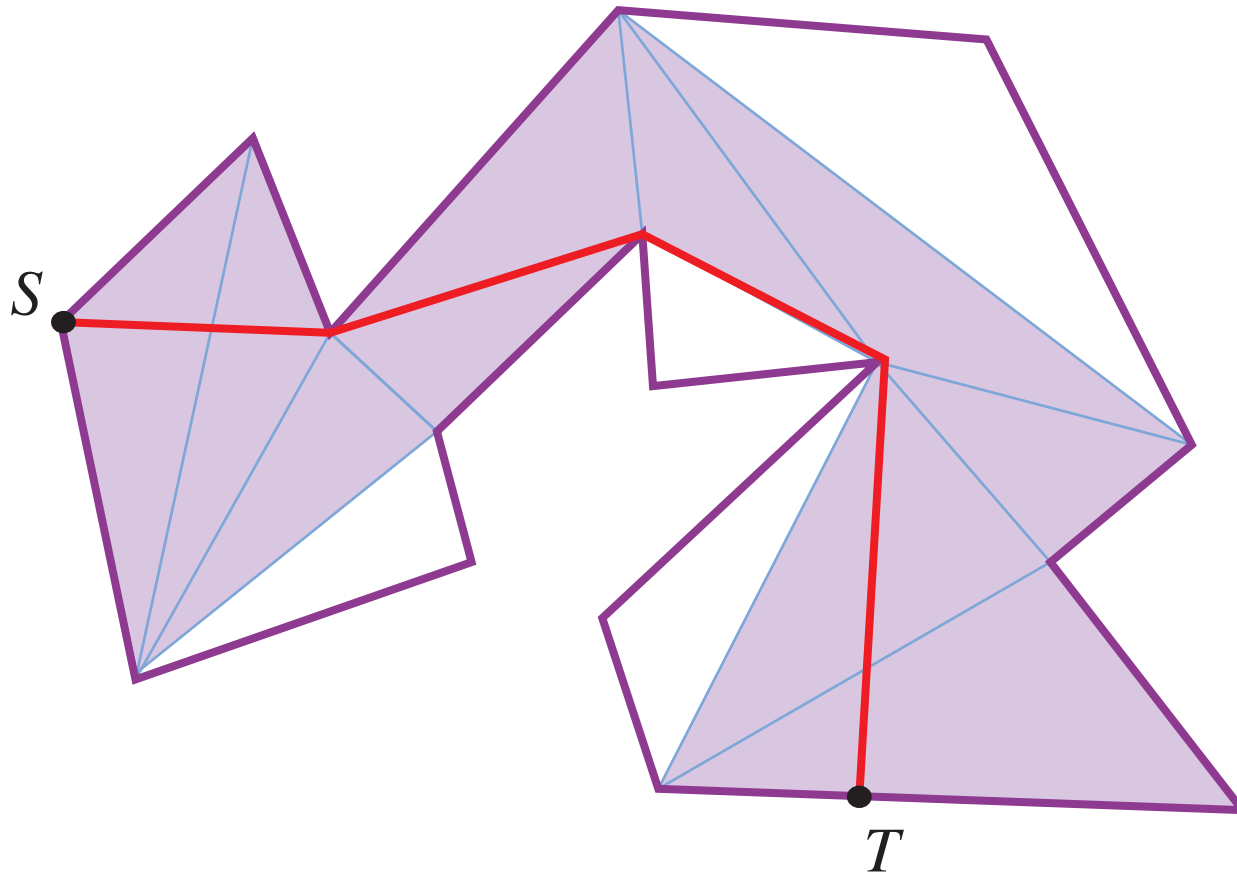
Geometric Shortest Paths -- Polygon

Funnel Algorithm



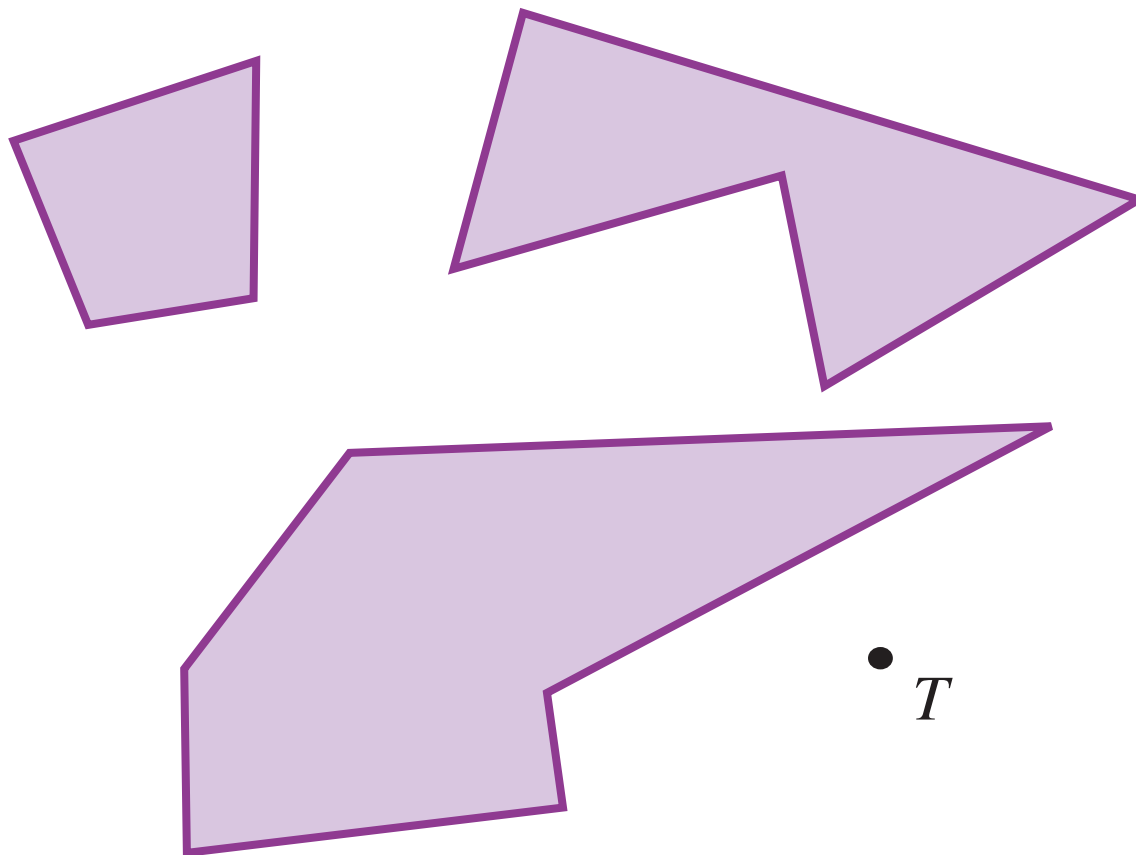
Geometric Shortest Paths -- Polygon

Funnel Algorithm -- $O(n)$ Guibas, Lee & Preparata, early '80's



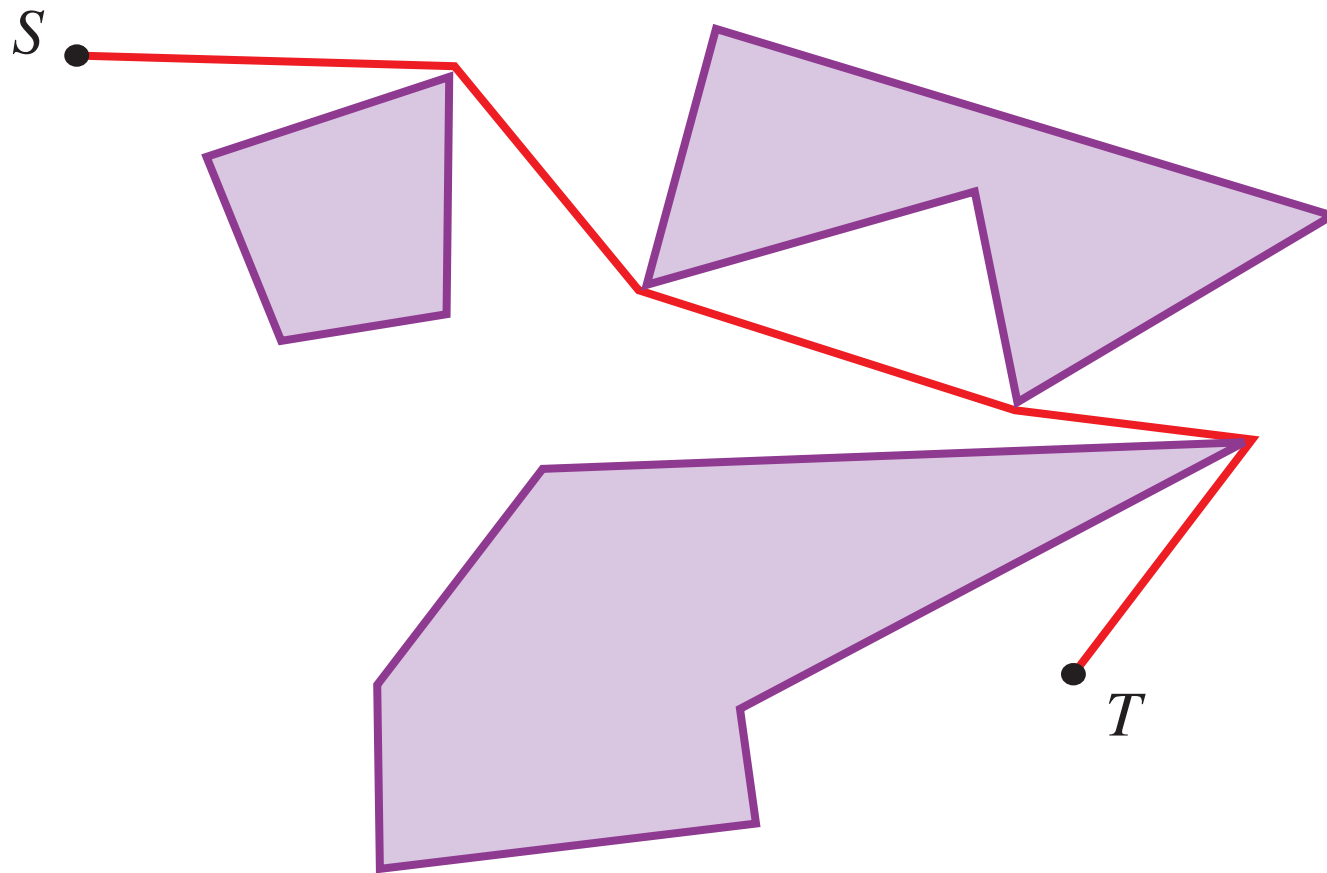
Geometric Shortest Paths -- Polygonal Domain

S •



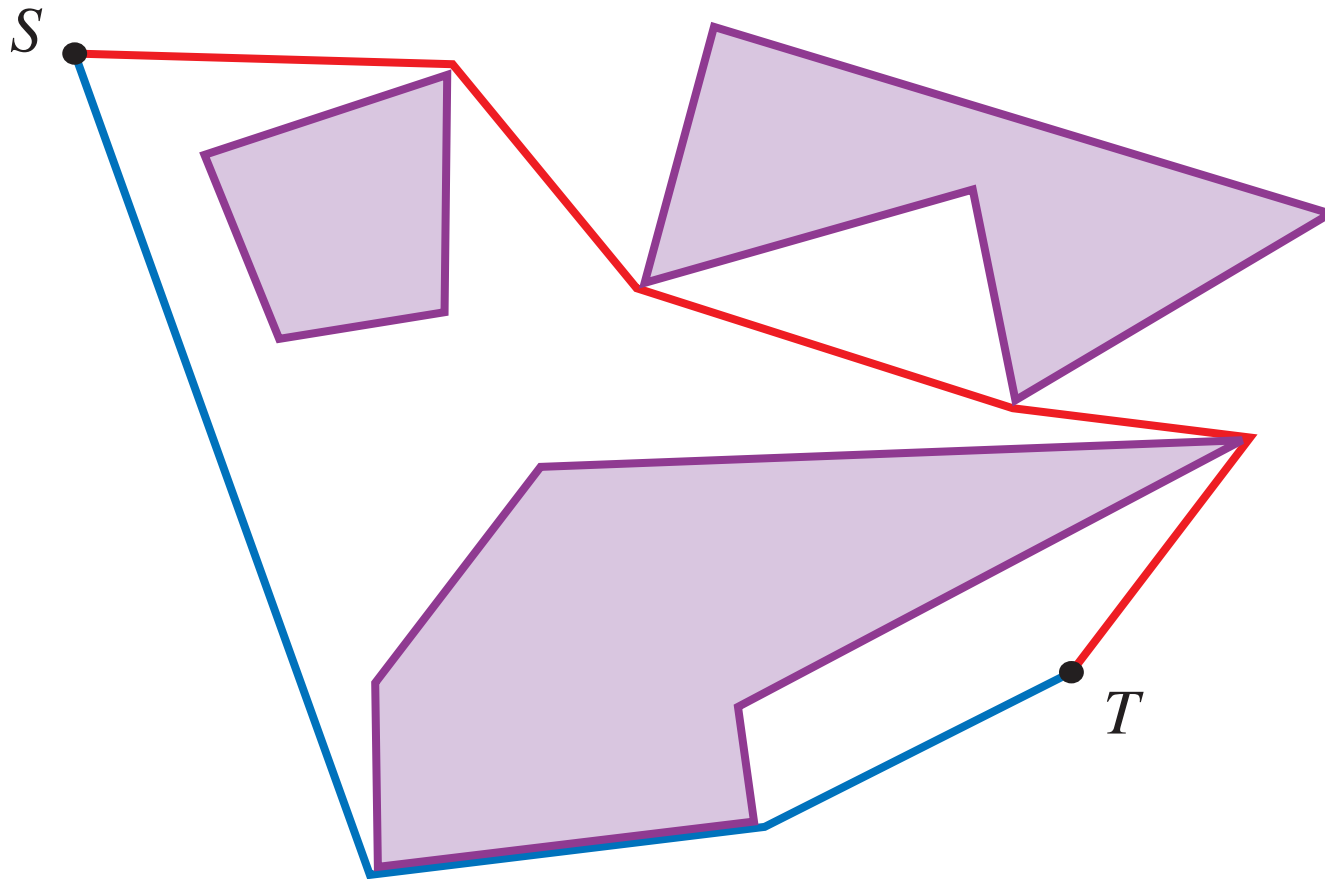
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Geometric Shortest Paths -- Polygonal Domain



multiple elastic band
solutions

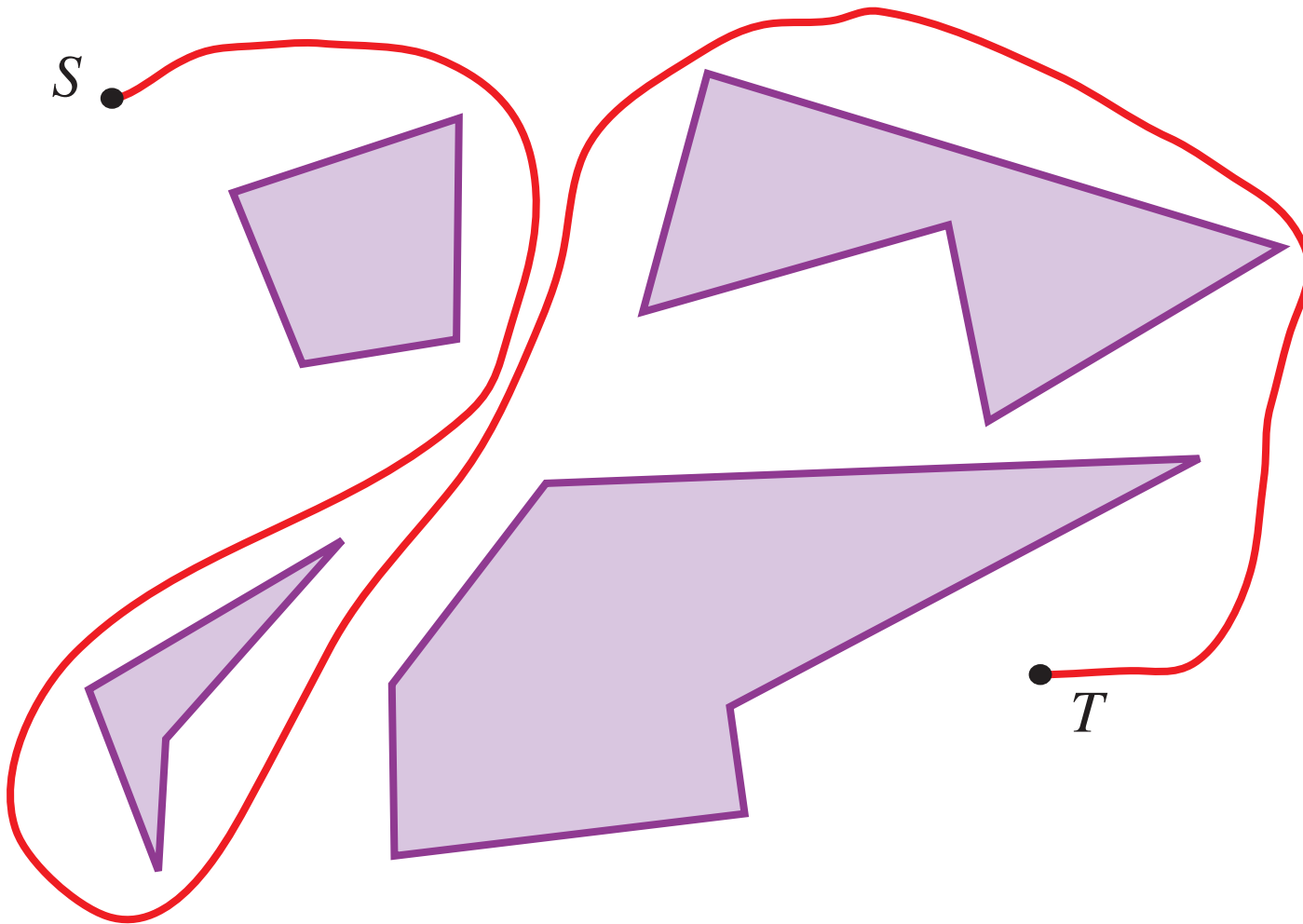
Geometric Shortest Paths -- Polygonal Domain



multiple elastic band
solutions

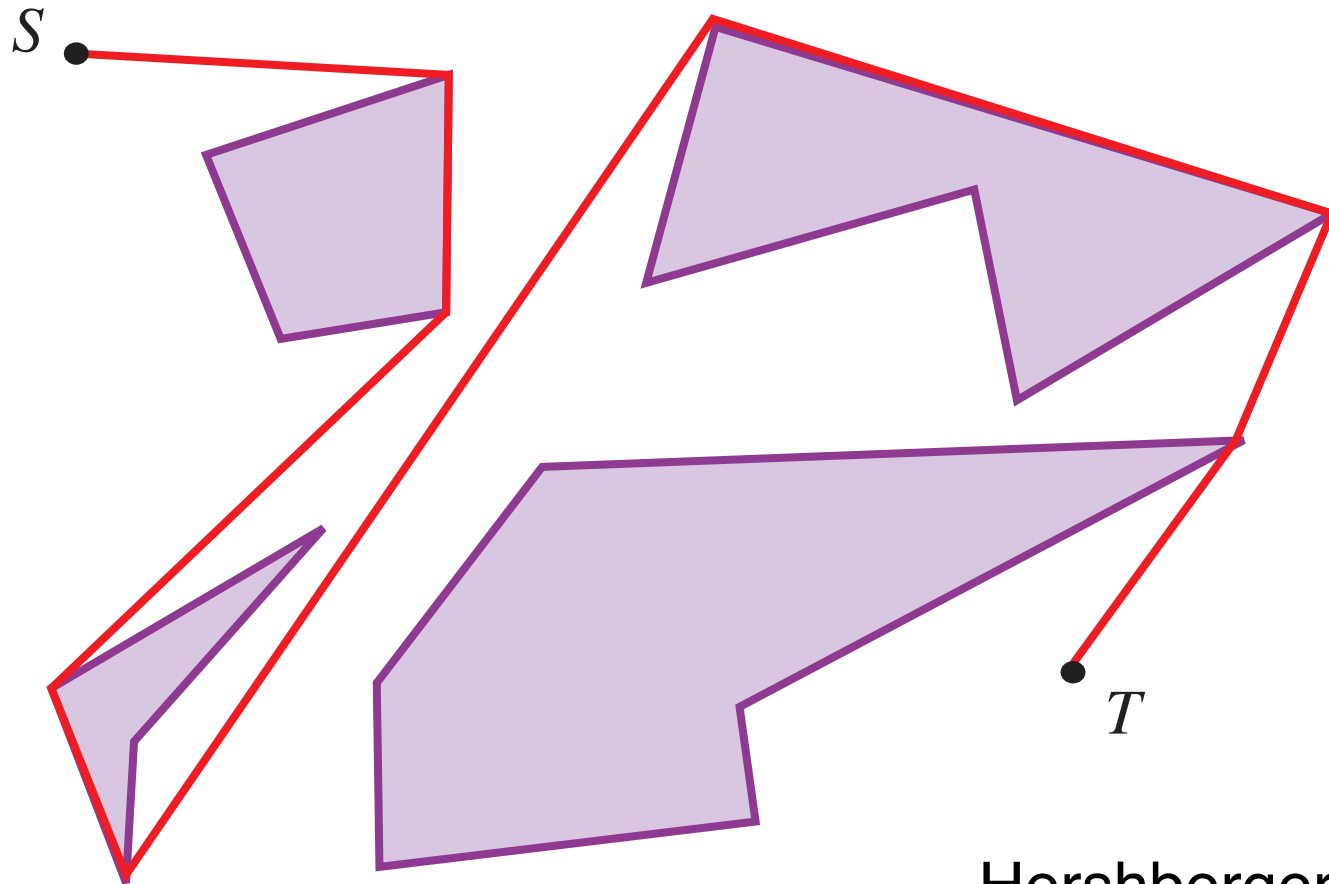
Geometric Shortest Paths -- Polygonal Domain

homotopic shortest path problem (shrinking an elastic band)



Geometric Shortest Paths -- Polygonal Domain

homotopic shortest path problem (shrinking an elastic band)

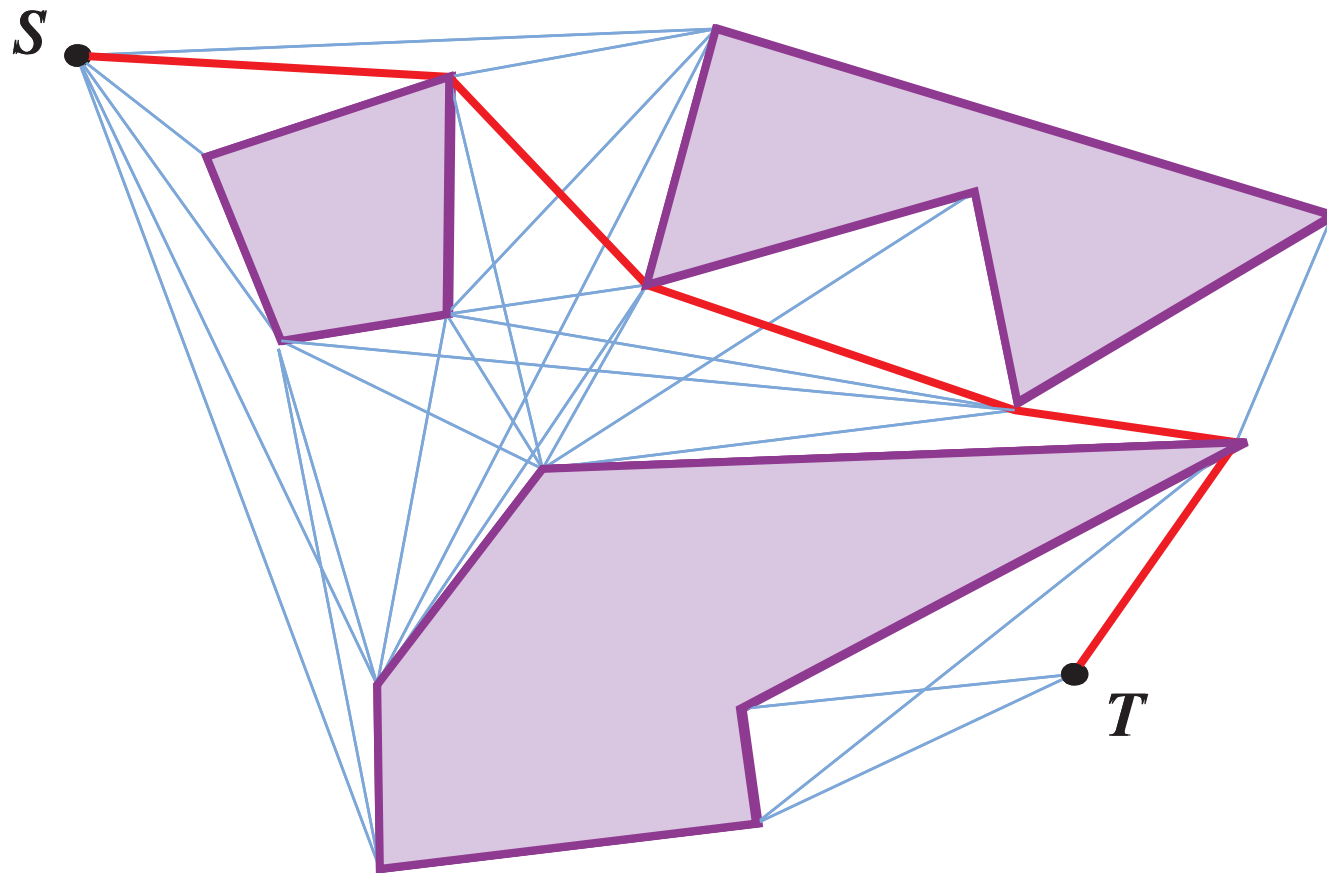


-- Hershberger & Snoeyink, '94

-- Efrat & Kobourov & Lubiw, '02

Geometric Shortest Paths -- Polygonal Domain

reducing to a graph problem



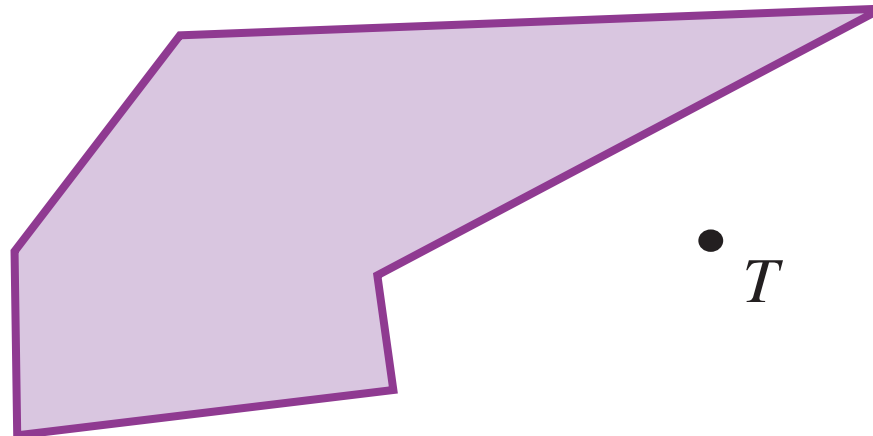
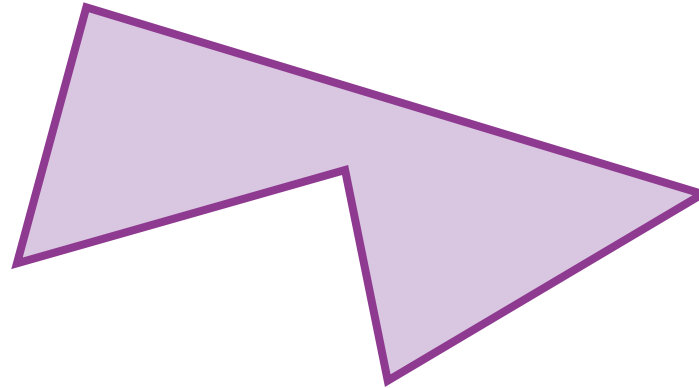
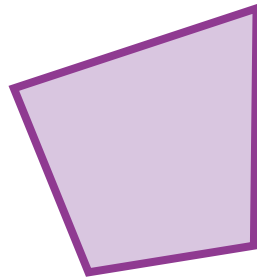
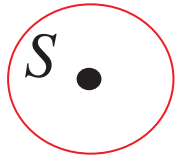
- construct **visibility graph**
- apply Dijkstra's graph algorithm

$$\left. \begin{array}{l} \text{construct visibility graph} \\ \text{apply Dijkstra's graph algorithm} \end{array} \right\} O(m + n \log n) = O(n^2)$$

Pocchiola & Vegter, Riviere, '95

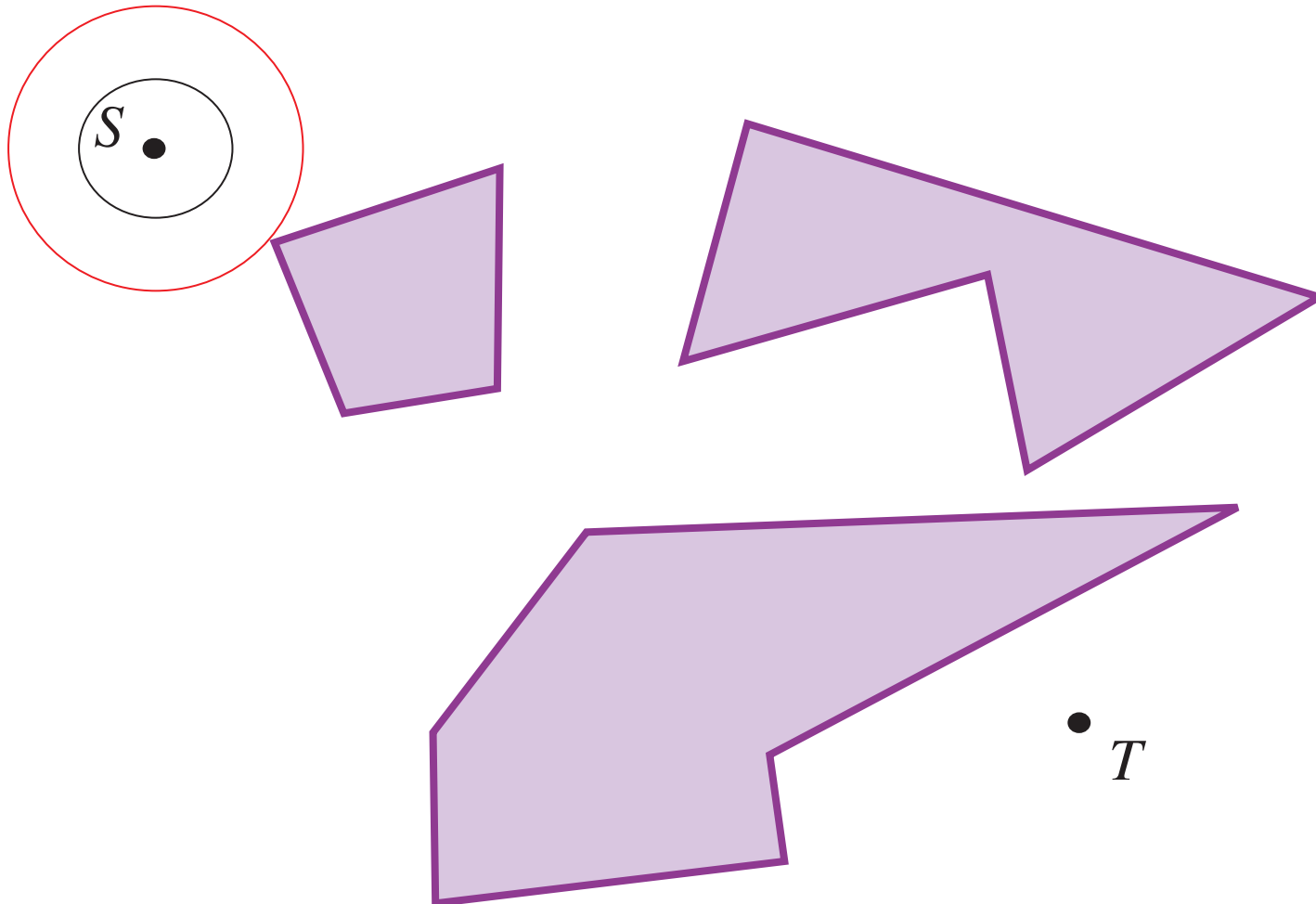
Geometric Shortest Paths -- Polygonal

Continuous Dijkstra



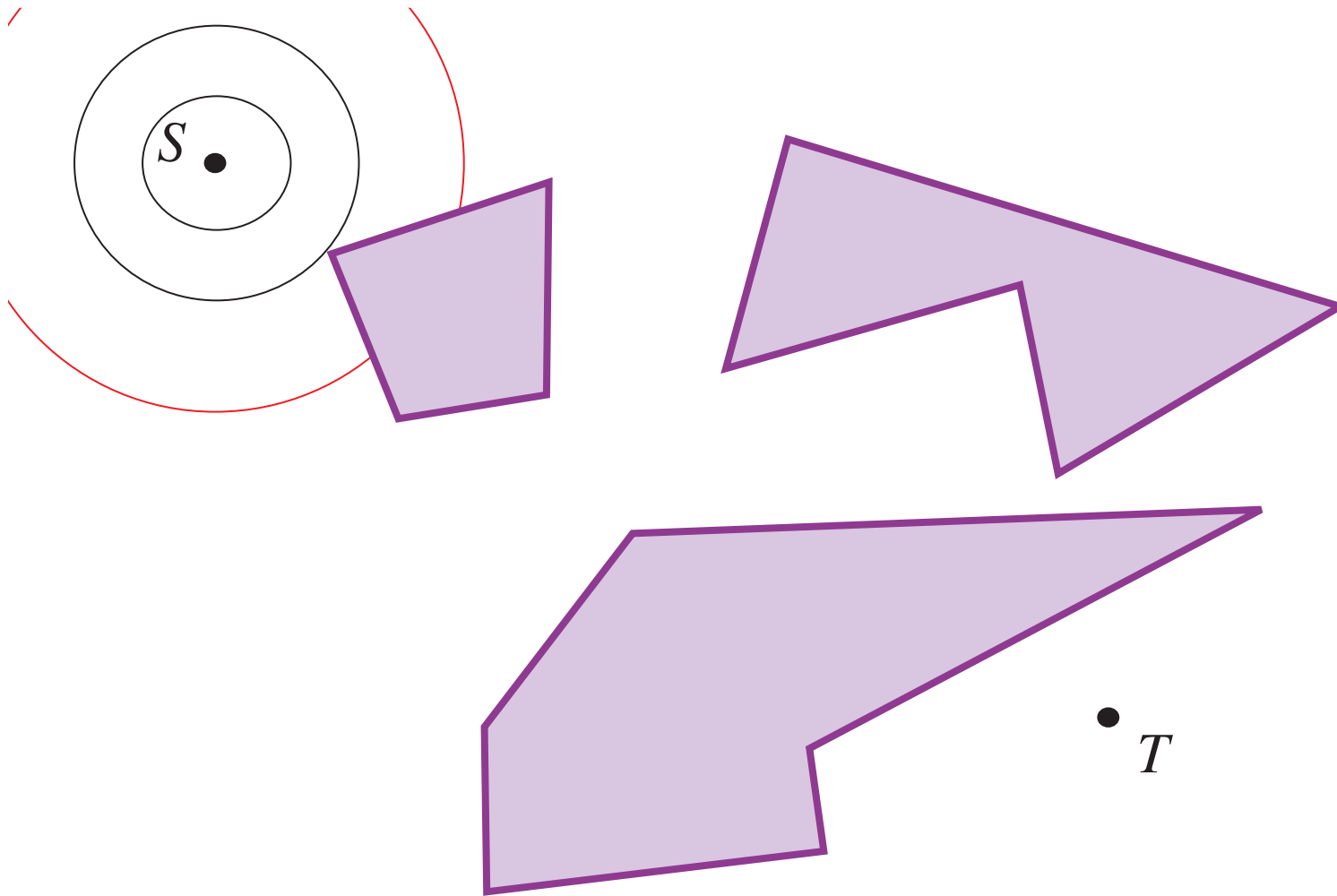
Geometric Shortest Paths -- Polygonal

Continuous Dijkstra



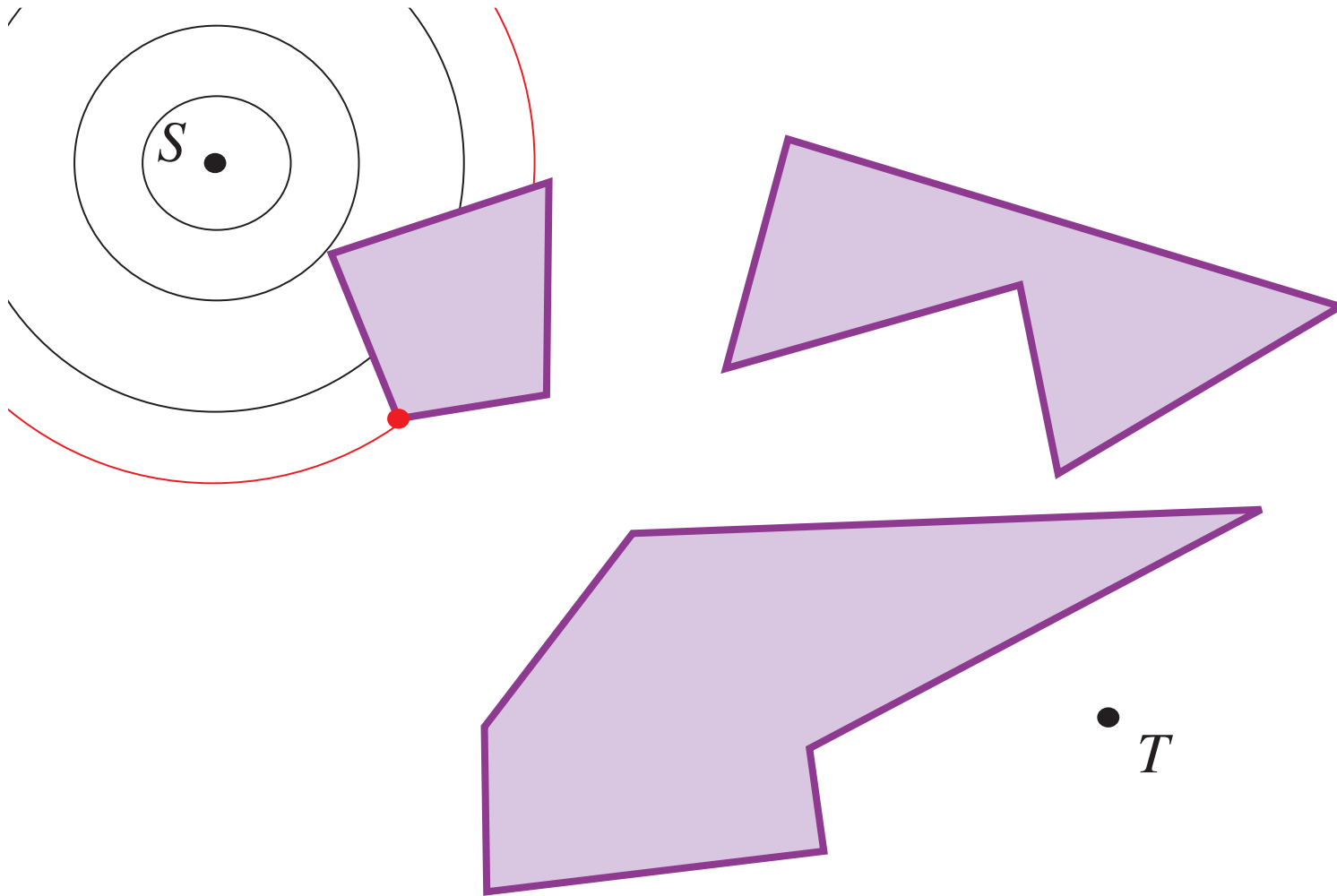
Geometric Shortest Paths -- Polygonal

Continuous Dijkstra



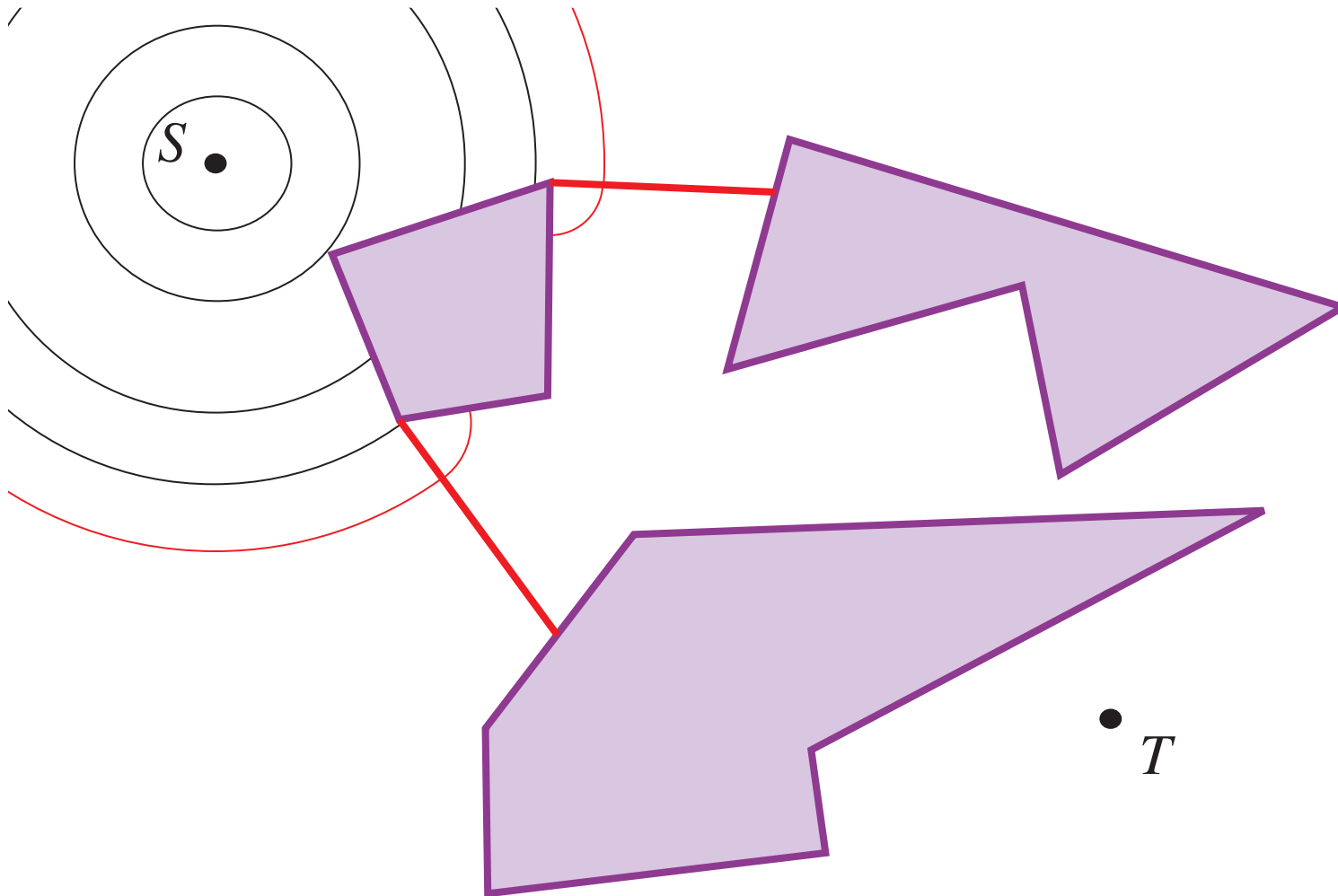
Geometric Shortest Paths -- Polygonal

Continuous Dijkstra



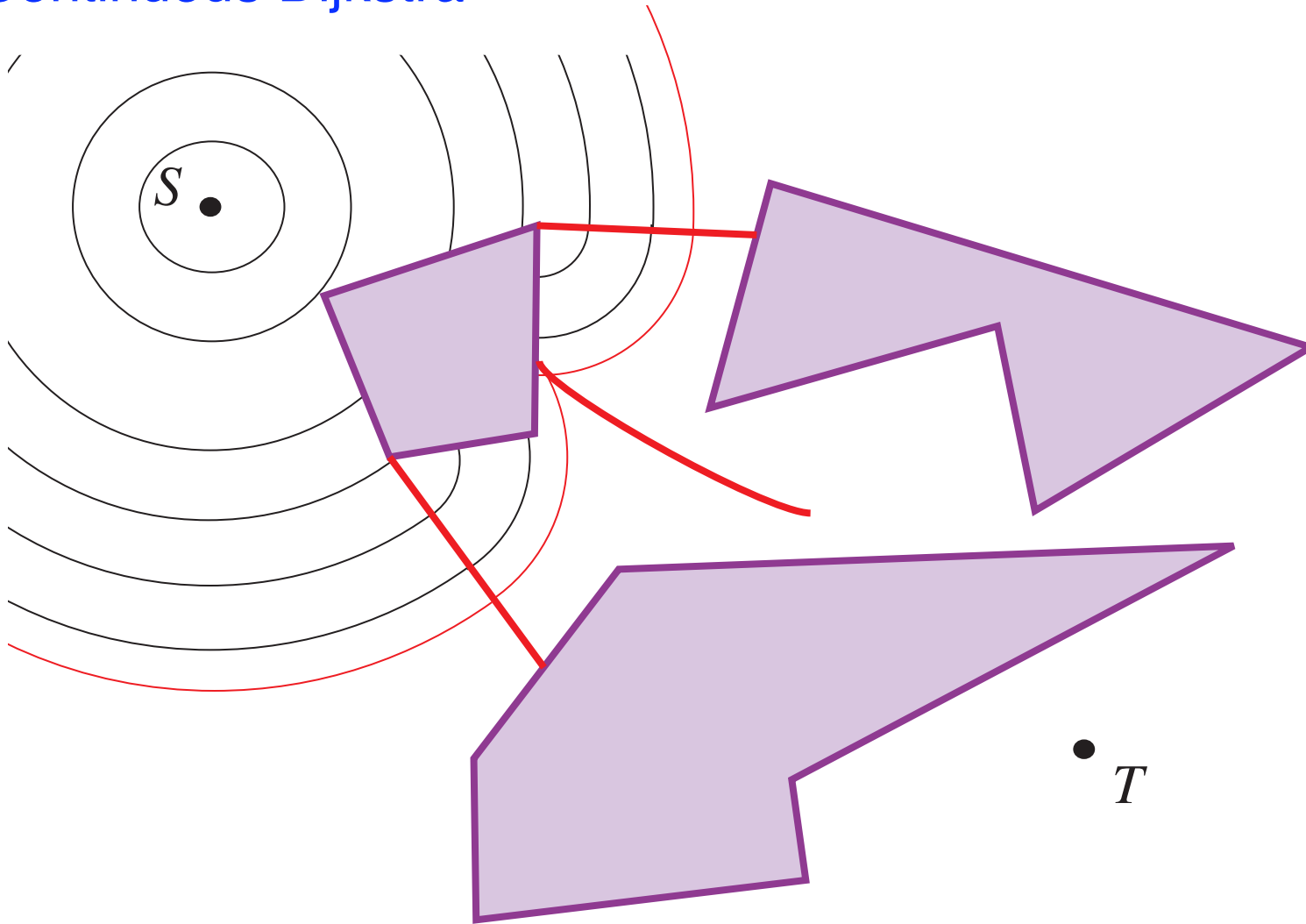
Geometric Shortest Paths -- Polygonal

Continuous Dijkstra



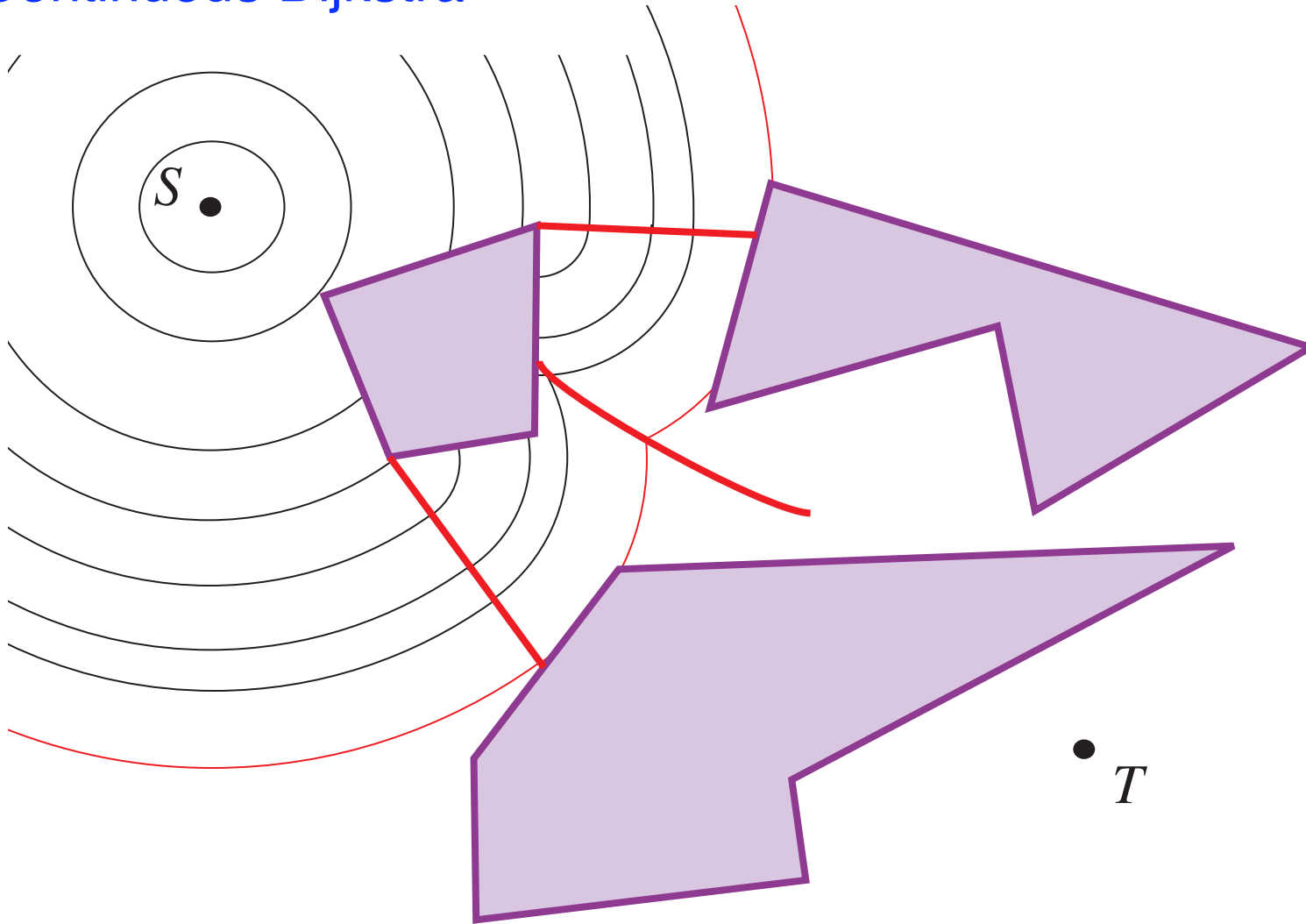
Geometric Shortest Paths -- Polygonal

Continuous Dijkstra



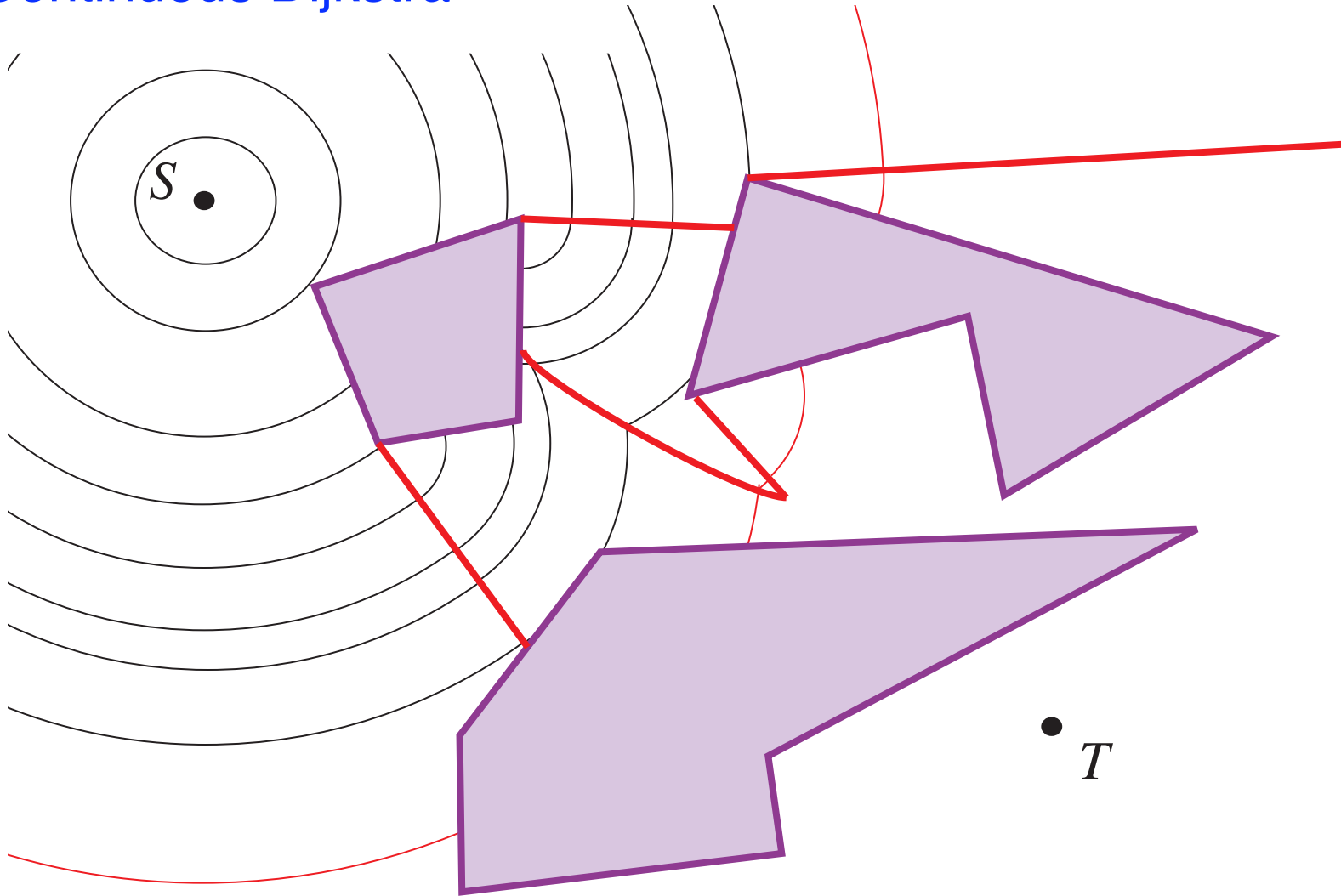
Geometric Shortest Paths -- Polygonal

Continuous Dijkstra



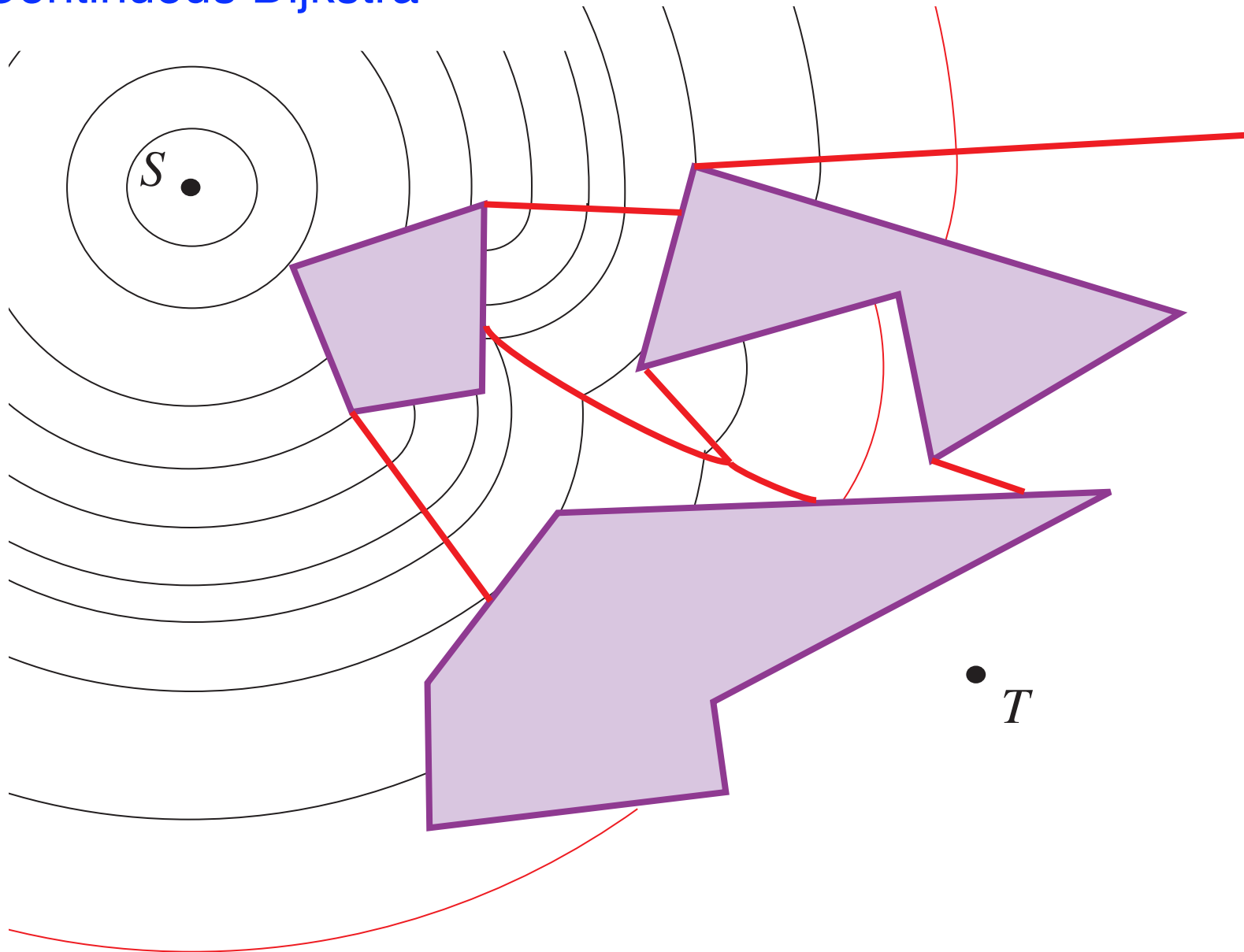
Geometric Shortest Paths -- Polygonal

Continuous Dijkstra



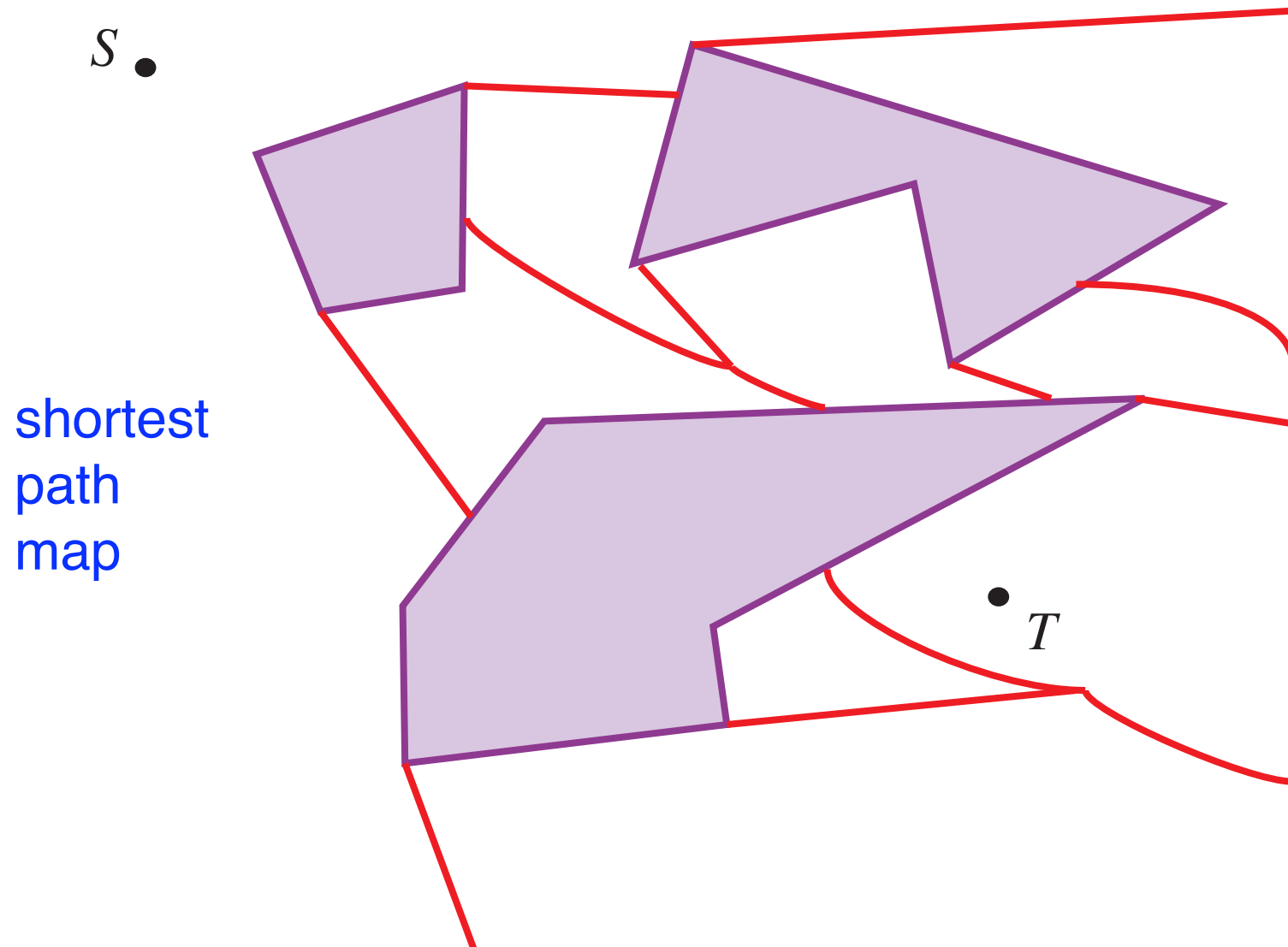
Geometric Shortest Paths -- Polygonal

Continuous Dijkstra



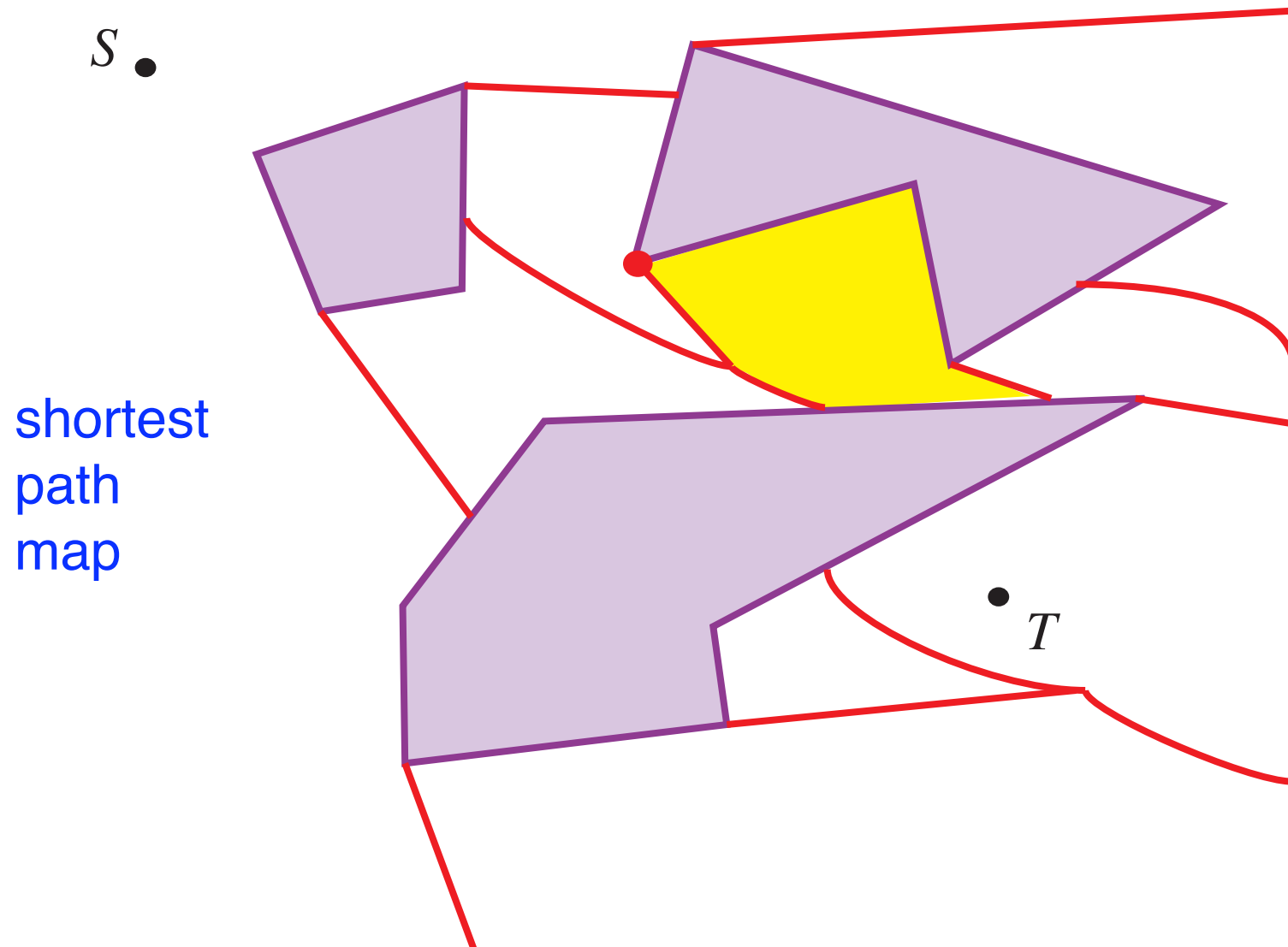
Geometric Shortest Paths -- Polygonal

Continuous Dijkstra



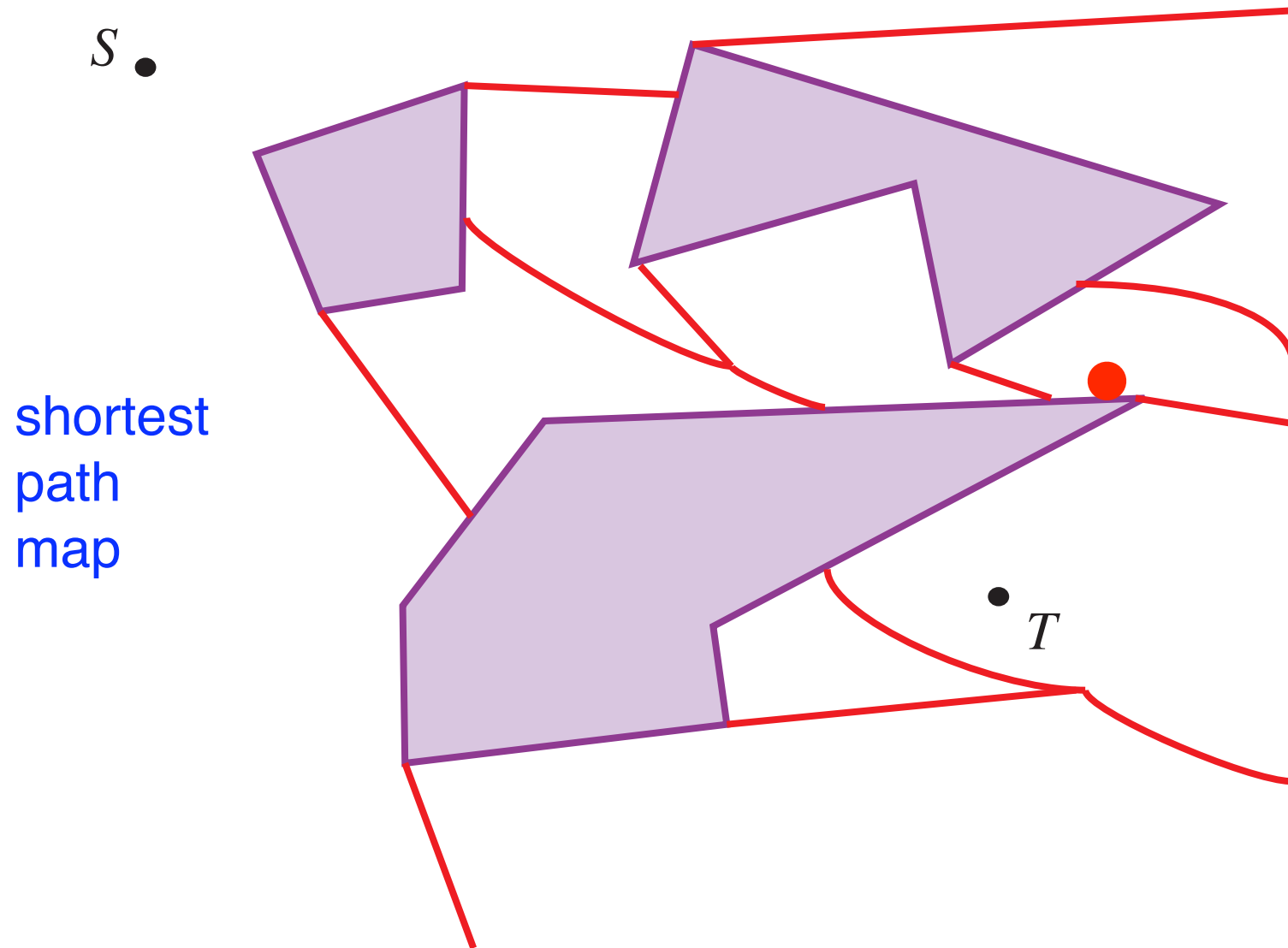
Geometric Shortest Paths -- Polygonal

Continuous Dijkstra



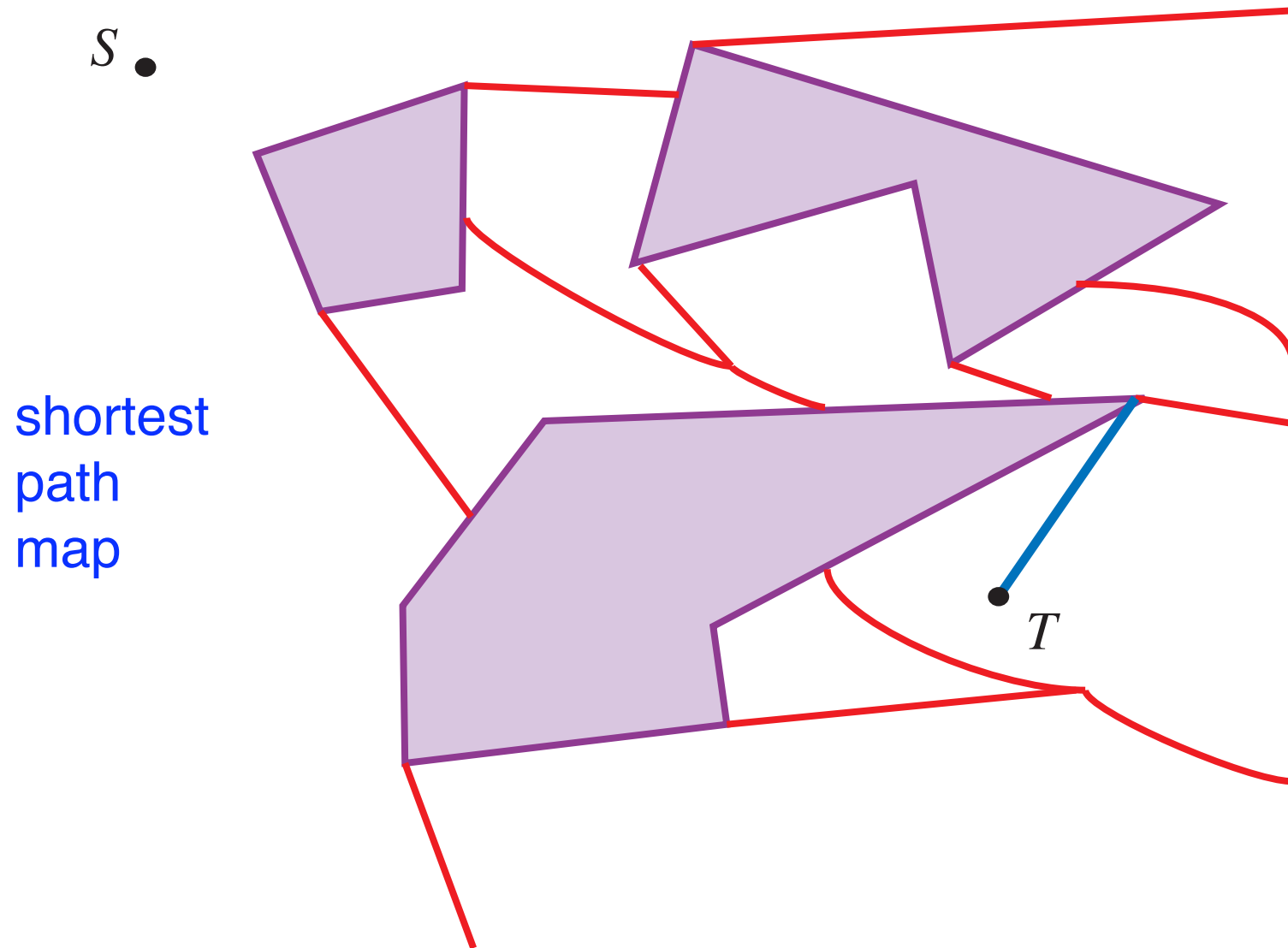
Geometric Shortest Paths -- Polygonal

Continuous Dijkstra



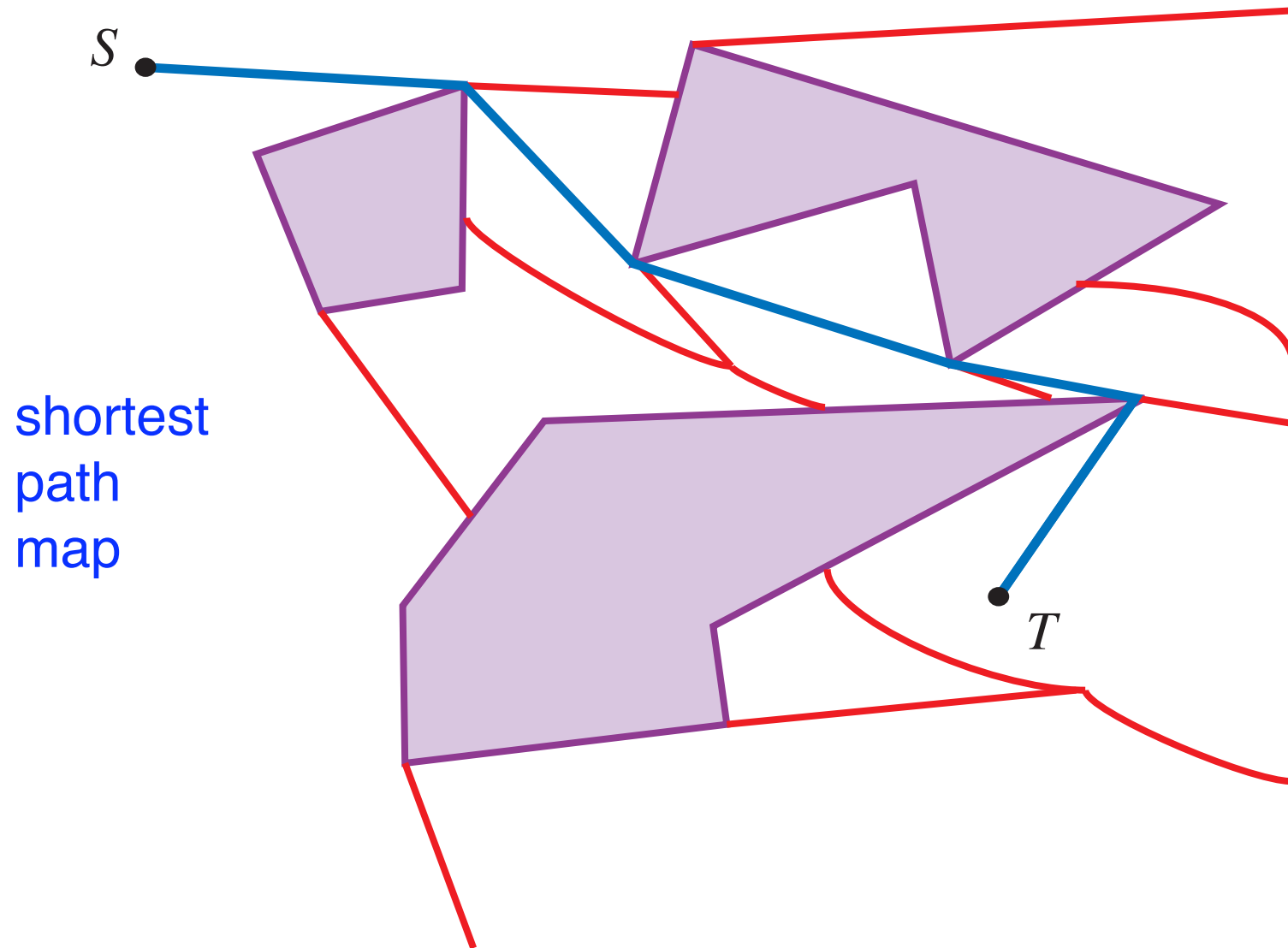
Geometric Shortest Paths -- Polygonal

Continuous Dijkstra



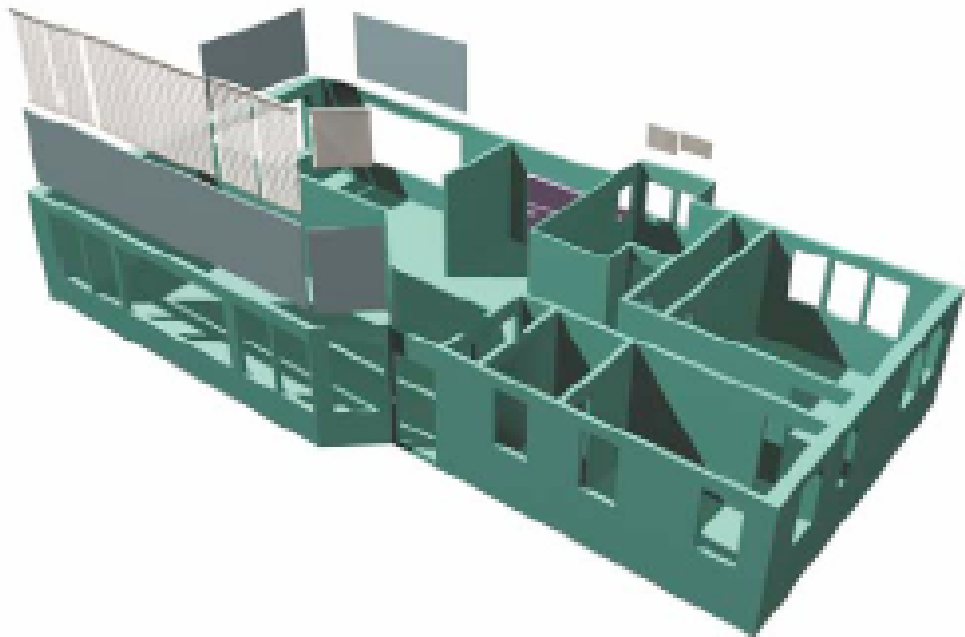
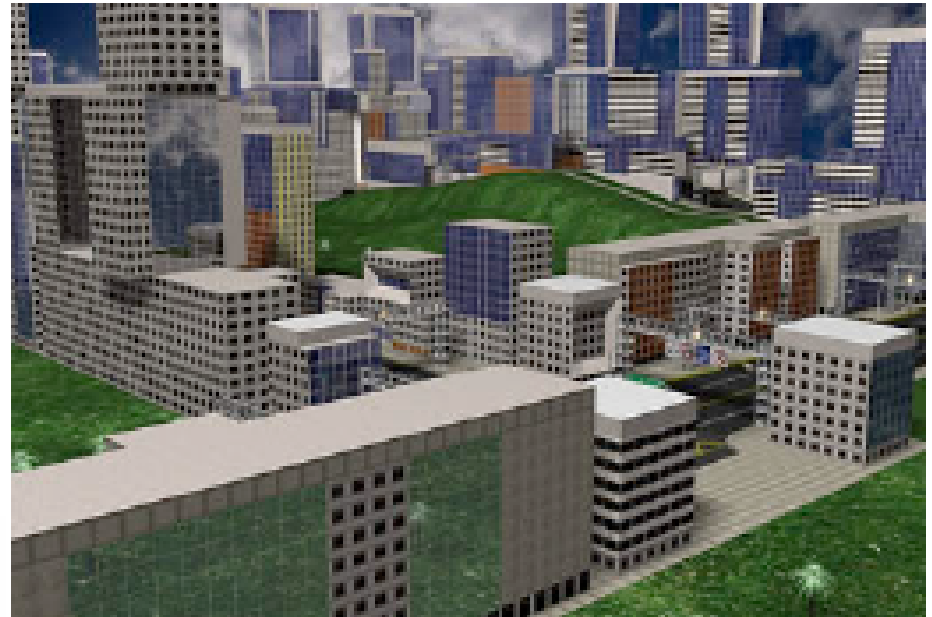
Geometric Shortest Paths -- Polygonal

Continuous Dijkstra - $O(n \log n)$ Mitchell, Hershberger & Suri, '93



3-D Shortest Path Problem

the general problem



3-D Shortest Path Problem

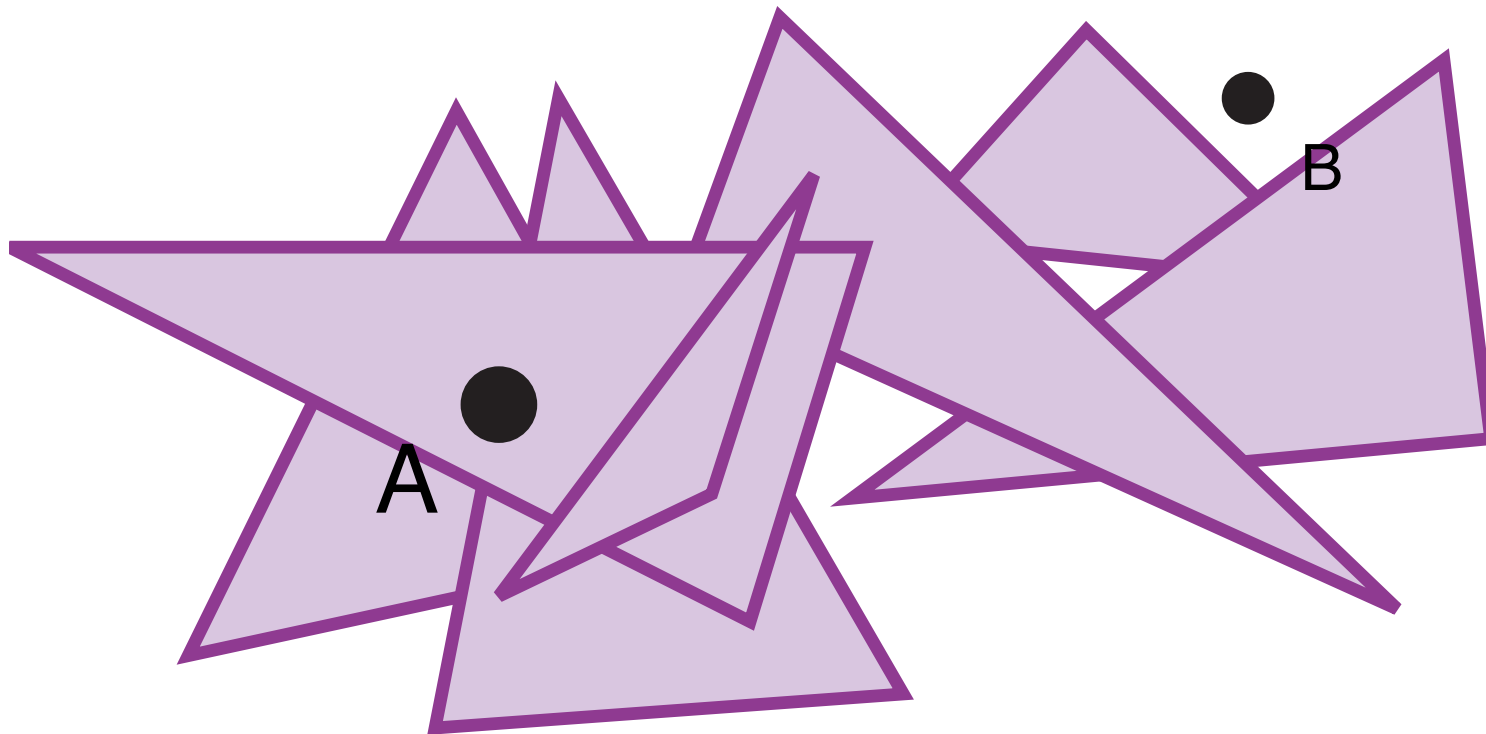
- NP-hard
- PSPACE algorithm, Canny '88
- approximation algorithms
- efficient algorithm for paths on polyhedral surfaces

3-D Shortest Path Problem

the general problem

NP-hard -- Canny & Reif, 1987

even for the case of parallel floating triangles

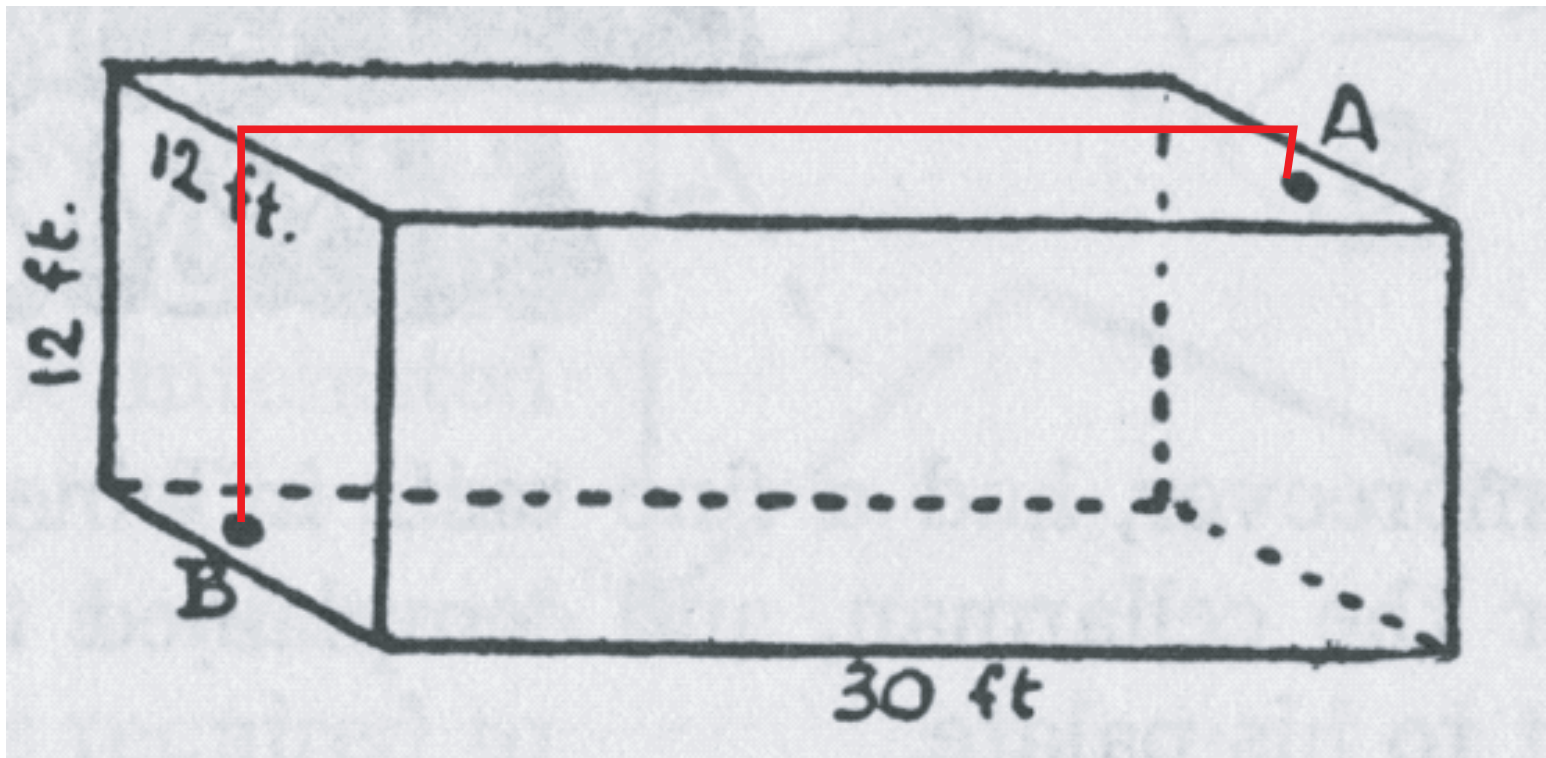


there are good approximation algorithms

Shortest Path Problem on a Polyhedral Surface

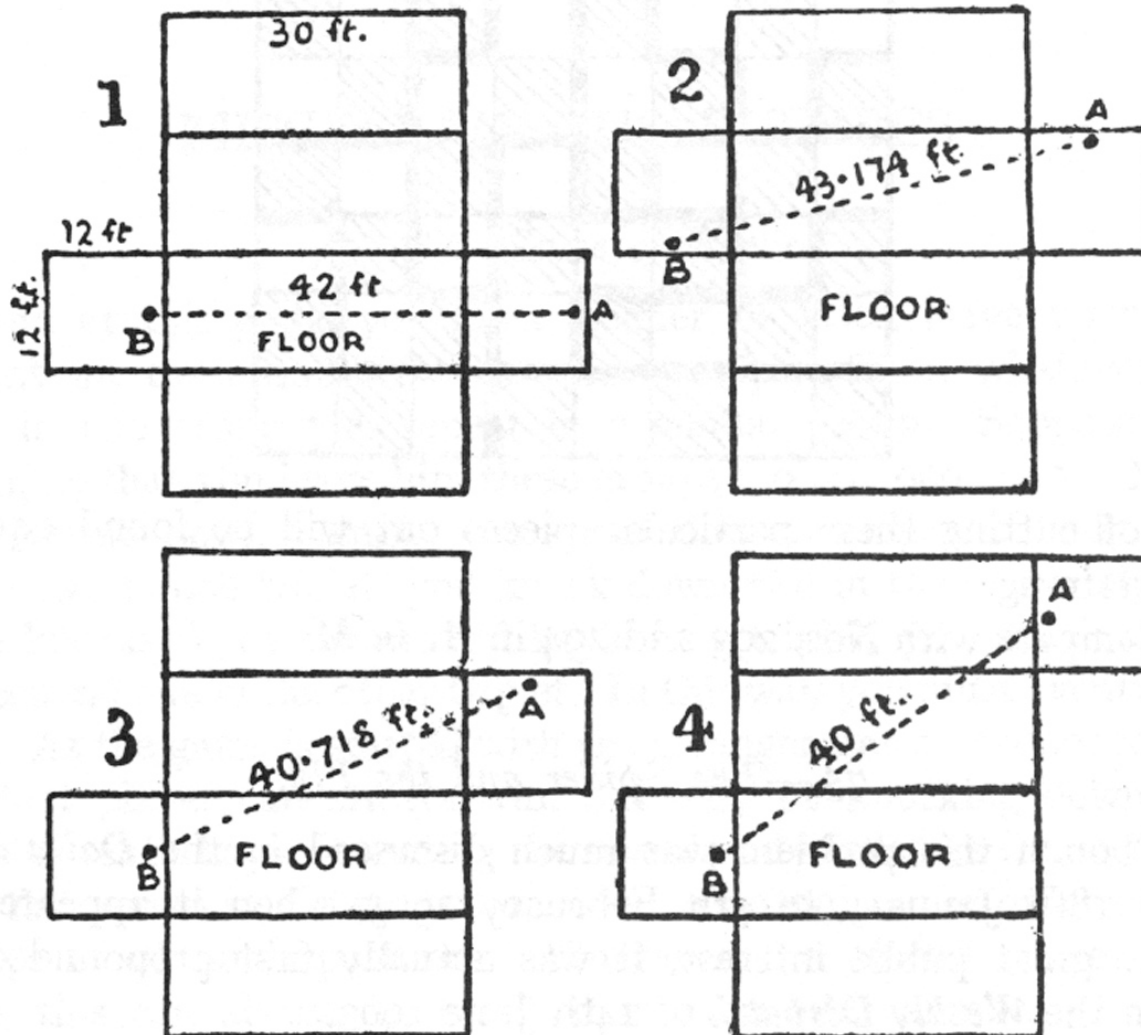
the spider and the fly problem

Dudeney, *The Canterbury Puzzles*, 1958



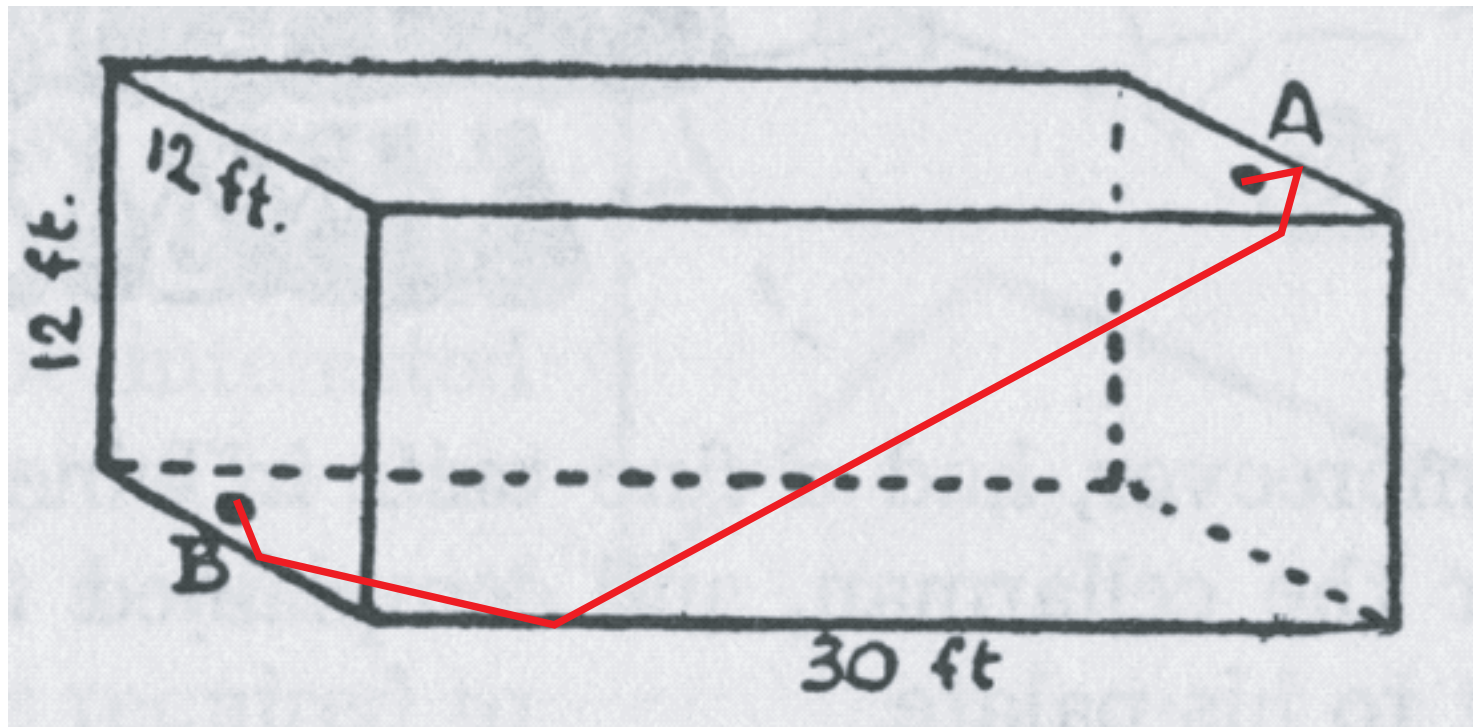
3-D Shortest Path Problem

the spider and the fly problem



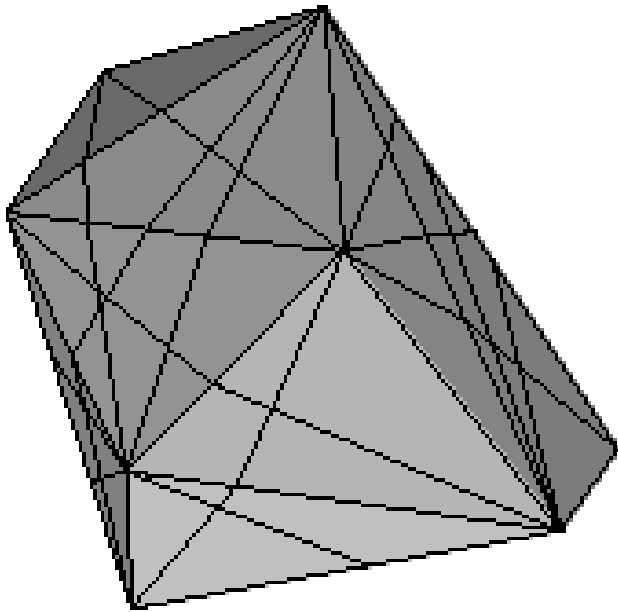
3-D Shortest Path Problem

the spider and the fly problem



3-D Shortest Path Problem

paths on polyhedral surfaces

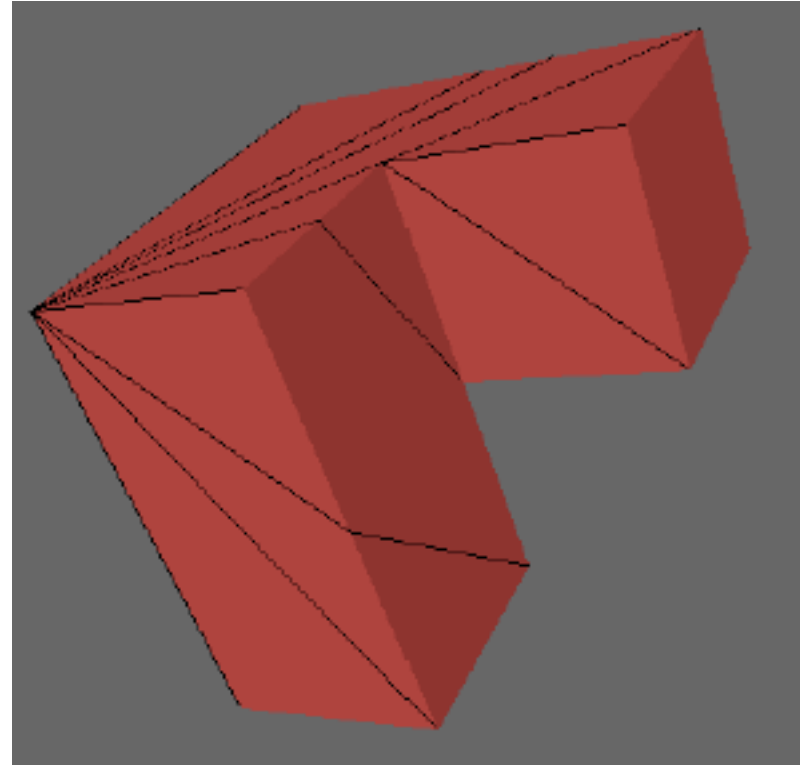
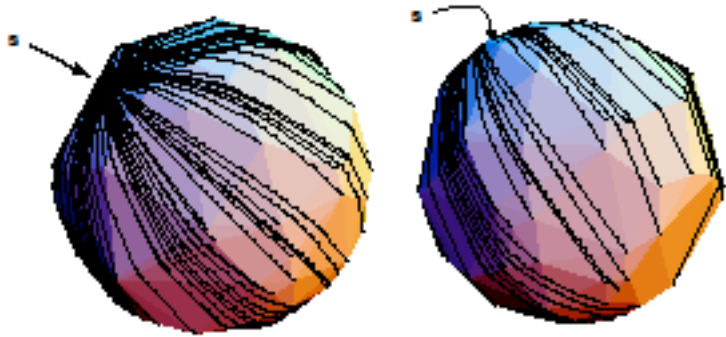


- $O(n^5)$ O'Rourke, Suri, Booth, '85
- $O(n^2)$ Chen, Han, '96
- $O(n \log^2 n)$ Kapoor, '99
- approximation algorithms

pictures from Kaneva & O'Rourke, '00

3-D Shortest Path Problem

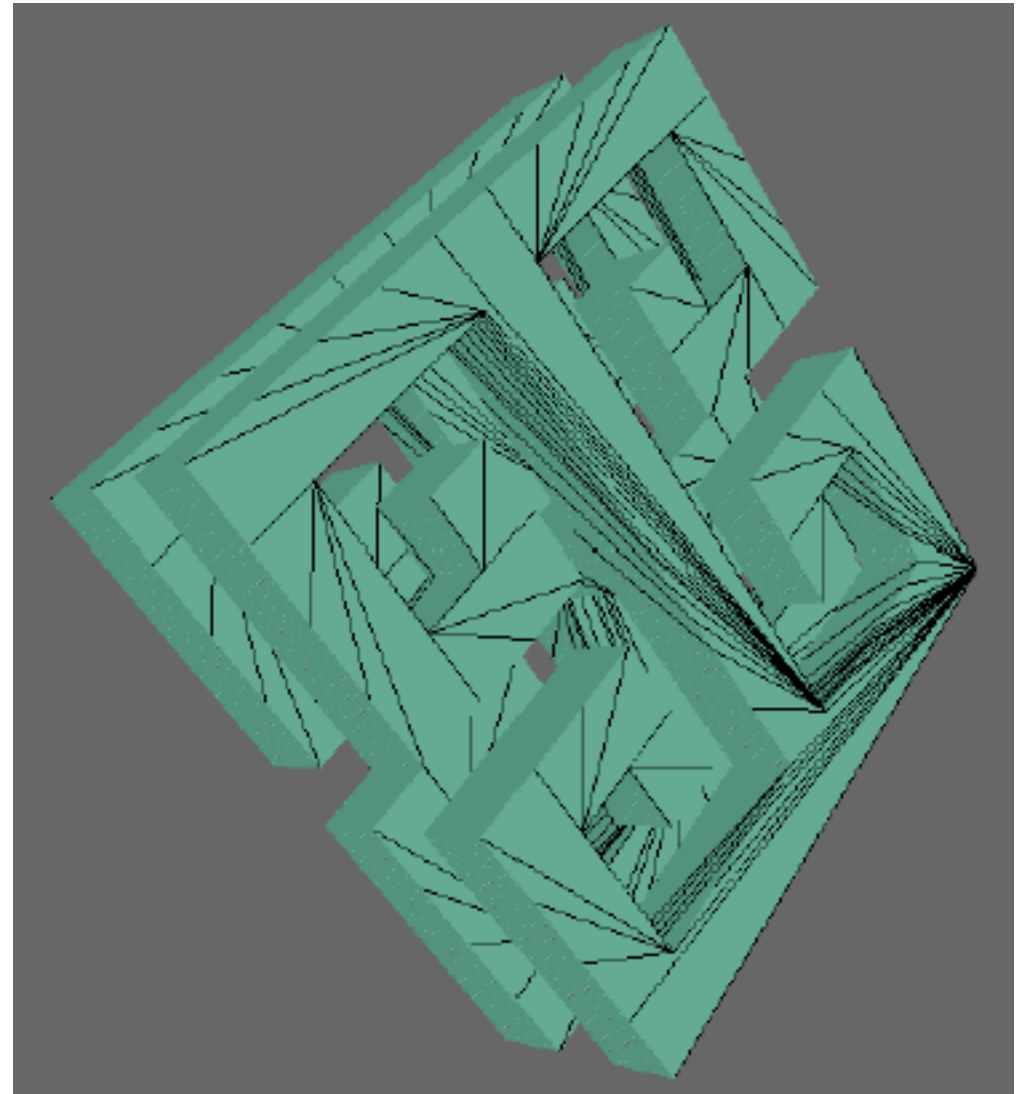
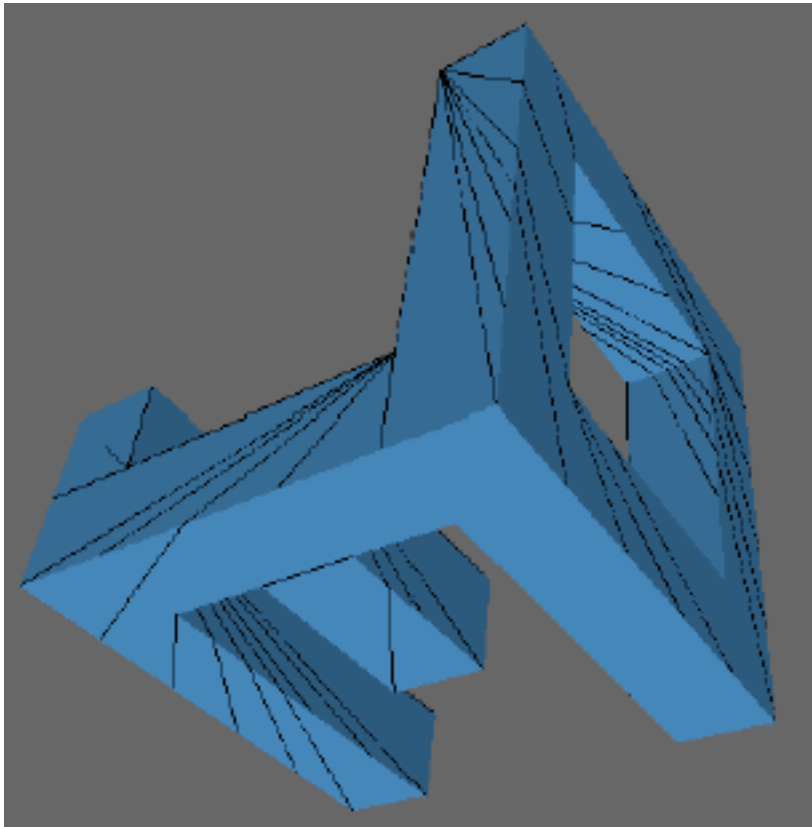
paths on polyhedral surfaces



pictures from Kaneva & O'Rourke, '00

3-D Shortest Path Problem

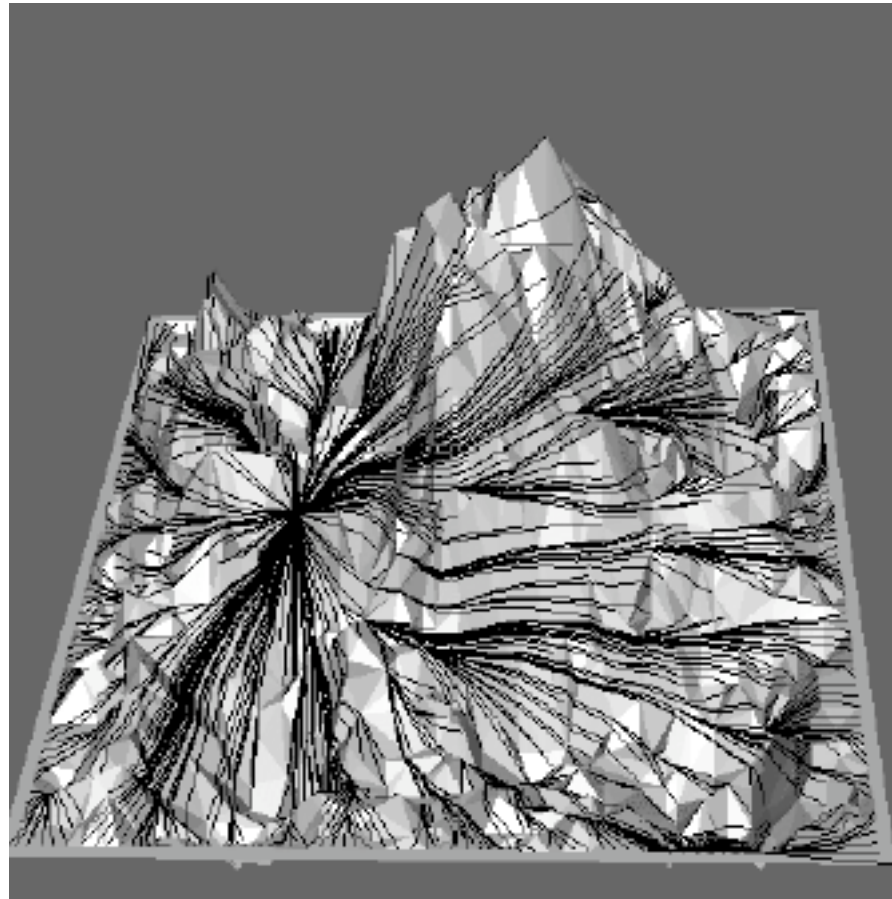
paths on polyhedral surfaces



pictures from Kaneva & O'Rourke, '00

3-D Shortest Path Problem

paths on polyhedral surfaces



pictures from Kaneva & O'Rourke, '00