Algorithms for Shortest Paths

course web page:  
https://cs.uwaterloo.ca/~alubiw/CS860.html

piazza:  
https://piazza.com/uwaterloo.ca/fall2014/cs860/home

Learn - for marks

desire path
Historical

Columbus

Anasazi
Topics

- Intro and Dijkstra's algorithm
- basic algorithms for geometric shortest paths
- basic algorithms for graph shortest paths

- more graph shortest path algorithms: all pairs, kth shortest, planar graphs, forbidden pairs, etc.
- more geometric shortest path algorithms: link distance, polyhedral surfaces, 3D, weighted region, etc.

- further topics
  - spanners
  - network routing
  - reconfiguration problems
  - touring (TSP)
  - and etc.
Topics

- basic algorithms for geometric shortest paths

Polygon

\( O(n) \)

fixed target versus query versions

Polygonal Domain

\( \Theta(n \log n) \)
Topics

- basic algorithms for graph shortest paths
  
  Dijkstra, Bellman-Ford

- more graph shortest path algorithms:
  
  - all pairs Timothy Chan
  - kth shortest David Eppstein, *Finding the k shortest paths*, cited by 1293
  - planar graphs highway graphs
  - forbidden pairs
  - etc. maybe disjoint paths
  
  computing diameter
more geometric shortest path algorithms:
- link distance
- polyhedral surfaces
\[ \frac{1}{2} D \]
- 3D
- weighted region
- etc.
Topics

- further topics
  - spanners - make graph less dense while approximately preserving distances
  - network routing
    - greedy routing
  - reconfiguration problems, diameter
  - touring (TSP)

- and etc.
Credit requirements

- 2 assignments (20%)
- presentation of 1 paper (30%) - 30 minutes + discussion
- exploration of open question (20%) - written report, approx. 2 pages. Formulate a good open question and suggest how to tackle it. BONUS if you succeed.
- mini reviews of presented papers (30%) - half page reports, submitted through Piazza.
Shortest Paths in Graphs

\[ |V| = n \quad |E| = m \]

Input: a graph \( G = (V, E) \) with weights \( w: E \rightarrow \mathbb{R} \) on edges, vertices \( s, t \)
Find the shortest [simple] path from \( s \) to \( t \).

do not repeat vertices

NP-complete in general
- reduction from Hamiltonian path from \( s \) to \( t \)
  - set all edge weights to \(-1\)
  - ask for path of weight \( \leq -(n-1) \)

Difficulty is negative weight cycles.

polynomial time for
- directed acyclic graphs — topological sort
- non-negative weights — Dijkstra's
- graphs with no negative weight cycle — Bellman-Ford, dynamic programming.
Dijkstra's Shortest Path Algorithm

Idea

- Shortest path known
- Update step:
  - Initialize $S = \emptyset$, $d(s) = 0$, $d(u) = \infty \forall u \in V - S$
  - Pick $v \in V - S$ of $\min d(u)$
  - Add $v$ to $S$
  - Update $u \in V - S$, $d(u) = \min \{d(u), d(v) + w(v, u)\}$ for edge $(v, u)$

Correctness

Implementations

- Basic: $O(n^2 + m)$
- Heap: $O(n \log n + m)$
- Fibonacci heap: $O(n \log n + m)$

Total work on updates $O(m)$
Dijkstra’s paper

**A note on two problems in connexion with graphs**  
EW Dijkstra - Numerische mathematik, 1959 - Springer

For some history, see:  
**The Quest for the Shortest Route**  
C Demetrescu, GF Italiano - The Power of Algorithms, 2013 - Springer

Dijkstra:

At the time, algorithms were hardly considered a scientific topic. I wouldn’t have known where to publish it… The mathematical culture of the day was very much identified with the continuum and infinity. Could a finite discrete problem be of any interest? The number of paths from here to there on a finite graph is finite; each path is a finite length; you must search for the minimum of a finite set. Any finite set has a minimum – next problem, please. It was not considered mathematically respectable…

Read his paper on webpage and Piazza  
What is 2nd problem?  
Optional - find your favorite presentation of Dijkstra’s alg.
other implementations of Dijkstra’s algorithm

Dial’s method

for integer weights in $[0, C]$,


use buckets $0 \cdots nC$

max length of shortest paths

to store $d(w)$

just scan once through buckets $O(nC + m)$

total work for updates.

finding min
other implementations of Dijkstra’s algorithm

Dial’s method

reducing space from $nC$ to $C+1$

At any point in alg. $d(u)$ values, $u \in V \setminus S$
lie in range $\min \ldots \min + C$ — not counting $u$ with $d(u) = \infty$

So use cyclic buckets — space $C+1$

Pf. true initially when $S' = \{s\}$

update increases $\min$

$u$ with finite $d(u)$ — decreases, so still $\leq \min + C$

$u$ with $d(u) = \infty$

$$d(u) = \min \{ \min \{ d(v) \}, \min \{ d(v) + w(v, u) \} \} \leq \min + C$$

$\min$ $\leq C$

-time: $O(m + nC)$
other implementations of Dijkstra’s algorithm

double bucket method


"lower buckets" - K of them, initially 0...K-1, K determined later
hold single values

"upper buckets" - each bucket holds K values
# upper buckets = \frac{nC}{K}

Find min
- if lower buckets all empty then
  search upper buckets
  move (distribute) contents of first non-empty bucket to lower bucket
- search lower buckets.

Time \(O\left(\frac{nC}{K} + nK + m\right)\)
min with \(K = \sqrt{c}\)
\(O\left(n\sqrt{c} + m\right)\)
related papers to present

**Faster algorithms for the shortest path problem**
RK Ahuja, K Mehlhorn, J Orlin, RE Tarjan - Journal of the ACM (JACM), 1990 - dl.acm.org
Cited by 551
Related articles All 19 versions Cite Save

From: http://scholar.google.ca/scholar?q=Faster+Algorithms+for+the+Shortest+Path+Problem&btnG=&hl=en&as_sdt=0%2C5

**Undirected single-source shortest paths with positive integer weights in linear time**
M Thorup - Journal of the ACM (JACM), 1999 - dl.acm.org
Cited by 307
Related articles All 11 versions Cite Save


**Shortest paths algorithms: Theory and experimental evaluation**
BV Cherkassky, AV Goldberg, T Radzik - Mathematical programming, 1996 - Springer
Cited by 781
Related articles All 36 versions Cite Save

From: http://scholar.google.ca/scholar?q=Shortest+paths+algorithms%3A+Theory+and+experimental+evaluation&btnG=&hl=en&as_sdt=0%2C5

**Computing the shortest path: A* search meets graph theory**
AV Goldberg, C Harrelson - Proceedings of the sixteenth annual ACM- ..., 2005 - dl.acm.org
Cited by 462
Related articles All 20 versions Cite Save

From: http://scholar.google.ca/scholar?q=Computing+the+Shortest+Path%3A+A%2A+Search+Meets+Graph+Theory&btnG=&hl=en&as_sdt=0%2C5

**Trans-dichotomous algorithms for minimum spanning trees and shortest paths**
Cited by 282
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