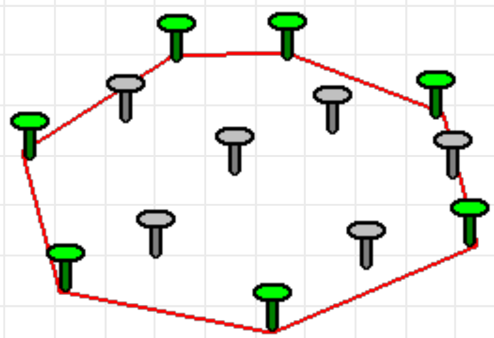
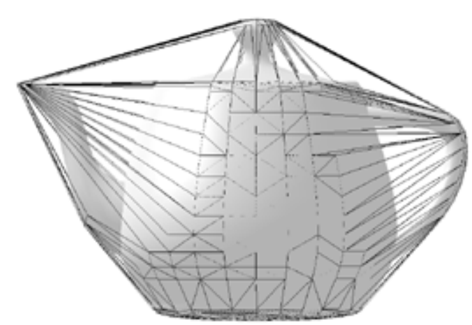
Given points in d-dimensional space, find a good “container” = convex polytope.
Many applications, e.g. collision detection, pattern recognition, motion planning . . .

In 2D, imagine putting a rubber band around the points



https://brilliant.org/wiki/convex-hull/

In 3D, wrap with shrink-wrap



Newton Collision Convex Hull

More formally:

A set is *convex* if for every two points *p*, *q* in the set, all points on the line segment *pq* are also in the set.

The *convex hull* of set *S* is the intersection of all convex sets that contain *S*.

Note that the convex hull of S is convex.
The fact that the convex hull of a set of points S is a convex polytope whose vertices are points of S requires a proof, which we will do later.

**Convex Hull Algorithms in 2D**

Almost any algorithmic paradigm will work, so this problem is a great one for Algorithms courses.

**Incremental Algorithm** — add points one by one in sorted order by x coordinate

**Graham's Algorithm**

Another sorting-base approach.
Sort the points radially around some point X inside the convex hull.

**Divide and Conquer Algorithm**

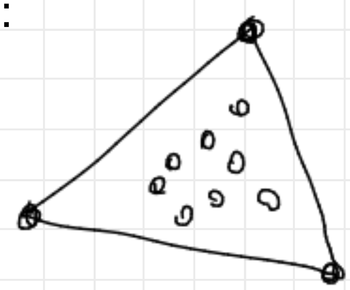
Divide the points in two by a vertical line (easy if we sort by x coordinate).
Recurse on each side.
Then combine the two sides.

**Lower Bound**

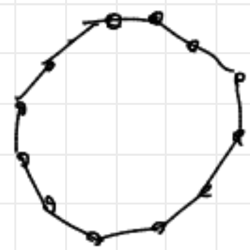
There is an Omega(n log n) lower bound on computing the ordered convex hull in 2D on a RAM with +,-,x.

**Output sensitive algorithm**

Idea:



should be faster than



Express the run time as a function of input size, $n$, and output size, $h$.

**Gift-Wrapping** (or "Jarvis's March")

So, what is the best algorithm in terms of $n$ and $h$?

O($n$ log $h$) algorithm — first developed by Kirkpatrick and Seidel 1986, uses linear time median finding.

improved by Timothy Chan, 1996.

**Chan's Algorithm**

Next: convex hull in higher dimensions.