Decisions with Multiple Agents: Game Theory

Alice Gao Lecture 24

Based on work by K. Leyton-Brown, K. Larson, and P. van Beek

By the end of the lecture, you should be able to

- Determine dominant-strategy equilibria of a 2-player normal form game.
- Determine pure-strategy Nash equilibria of a 2-player normal form game.
- Determine Pareto optimal outcomes of a 2-player normal form game.
- Calculate a mixed strategy Nash equilibrium of a 2-player normal form game.

Outline

Learning Goals

Prisoner's dilemma

Matching quarters

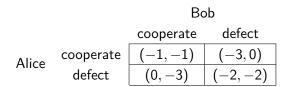
Dancing or concert?

Prisoner's dilemma

Matching quarters

Dancing or concert?

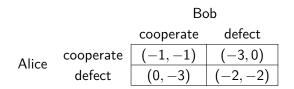
Prisoner's dilemma



CQ: Prisoner's dilemma - DSE

CQ: Which outcome, if any, is a dominant strategy equilibrium?

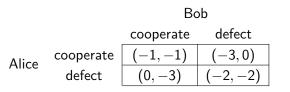
- (A) (cooperate, cooperate)
- (B) (cooperate, defect)
- (C) (defect, cooperate)
- (D) (defect, defect)
- (E) There is no dominant strategy equilibrium.



CQ: Prisoner's dilemma - NE

CQ: How many of the four outcomes are **pure-strategy Nash equilibria**?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4



CQ: Prisoner's dilemma - Pareto optimality

CQ: How many of the four outcomes are Pareto optimal?

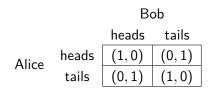
(A) 0 (B) 1 (C) 2 (D) 3 (E) 4 Bob Alice $\begin{array}{c} \text{Bob}\\ \text{cooperate} & \text{defect}\\ \text{defect} & (-1, -1) & (-3, 0)\\ (0, -3) & (-2, -2) \end{array}$

Prisoner's dilemma

Matching quarters

Dancing or concert?

Matching quarters

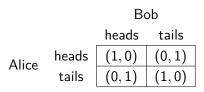


Alice wants the two coins to match whereas Bob wants the two coins to mismatch.

CQ: Matching quarters - NE

CQ: How many of the four outcomes are **pure-strategy Nash equilibria**?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

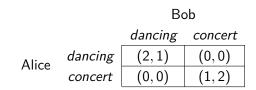


Prisoner's dilemma

Matching quarters

Dancing or concert?

Dancing or concert?



Alice and Bob want to sign up for an activity together. They both prefer to sign up for the same activity. However, Alice prefers dancing over going to a concert whereas Bob prefers going to a concert over dancing.

CQ: Why is a player willing to mix between two actions?

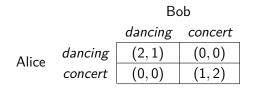
Consider a 2-player normal form game and fix Bob's strategy. Alice is willing to play heads 60% of the time and tails 40% of the time. Which of the following statements is true?

- (A) Alice's expected utility of playing heads is greater than her expected utility of playing tails.
- (B) Alice's expected utility of playing heads is less than her expected utility of playing tails.
- (C) Alice's expected utility of playing heads is same as her expected utility of playing tails.

CQ: Dancing or concert - mixed-strategy NE

CQ: At the mixed strategy Nash equilibrium, with what probability does **Alice go dancing**?

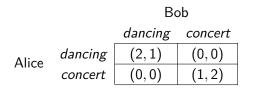
(A) 0 (B) 1/3 (C) 2/3 (D) 1



CQ: Dancing or concert - mixed-strategy NE

CQ: At the mixed strategy Nash equilibrium, with what probability does **Bob go dancing**?

(A) 0 (B) 1/3 (C) 2/3 (D) 1



Revisiting the Learning Goals

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