# The Value Iteration Algorithm 

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Based on work by K. Leyton-Brown, K. Larson, and P. van Beek

## Outline

## Learning Goals

Revisiting the Learning goals

## Learning Goals

By the end of the lecture, you should be able to

- Trace the execution of and implement the value iteration algorithm for solving a Markov Decision Process.


## CQ: Determine optimal action given true utility

CQ: What is the optimal action for state $s_{13}$ ?
(A) Up
(B) Down
(C) Left
(D) Right

$$
\begin{aligned}
U(s, a) & =\sum_{s^{\prime}} P\left(s^{\prime} \mid s, a\right) U\left(s^{\prime}\right) \\
\pi^{*}(s) & =\arg \max _{a} U(s, a) .
\end{aligned}
$$

The true utilities $U(s)$ are given below.

|  | 1 | 2 | 3 | 4 |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 0.705 | 0.655 | 0.611 | 0.388 |
| 2 | 0.762 | $X$ | 0.660 | -1 |
| 3 | 0.812 | 0.868 | 0.918 | +1 |

## CQ: Solve the Bellman equations efficiently

CQ: Can we solve the system of Bellman equations efficiently?
(A) Yes
(B) No
(C) I don't know

The Bellman equation for $U\left(s_{11}\right)$ :

$$
\begin{aligned}
& U\left(s_{11}\right)=-0.04+\gamma \max \left[0.8 U\left(s_{12}\right)+0.1 U\left(s_{21}\right)+0.1 U\left(s_{11}\right)\right. \\
& 0.9 U\left(s_{11}\right)+0.1 U\left(s_{12}\right) \\
& 0.9 U\left(s_{11}\right)+0.1 U\left(s_{21}\right) \\
&\left.0.8 U\left(s_{21}\right)+0.1 U\left(s_{12}\right)+0.1 U\left(s_{11}\right)\right]
\end{aligned}
$$

## CQ: Value iteration

CQ: What is $U_{1}\left(s_{23}\right)$ ?
(A) $(-\infty, 0)$
(B) $[0,0.25)$
(C) $[0.25,0.5)$
(D) $[0.5,0.75)$
(E) $[0.75,1]$
$U_{0}(s)$ :

|  | 1 | 2 | 3 | 4 |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 0 | 0 | 0 | 0 |
| 2 | 0 | $X$ | 0 | -1 |
| 3 | 0 | 0 | 0 | +1 |

## CQ: Value iteration

CQ: What is $U_{1}\left(s_{33}\right)$ ?
(A) 0.26
(B) 0.36
(C) 0.46
(D) 0.56
(E) 0.76
$U_{0}(s):$

|  | 1 | 2 | 3 | 4 |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 0 | 0 | 0 | 0 |
| 2 | 0 | $X$ | 0 | -1 |
| 3 | 0 | 0 | 0 | +1 |

## CQ: Value iteration

CQ: What is $U_{2}\left(s_{33}\right)$ ?
(A) 0.822
(B) 0.832
(C) 0.842
(D) 0.852
(E) 0.862

## CQ: Value iteration

CQ: What is $U_{2}\left(s_{23}\right)$ ?
(A) 0.464
(B) 0.466
(C) 0.468
(D) 0.470
(E) 0.472

## CQ: Value iteration

CQ: What is $U_{2}\left(s_{32}\right)$ ?
(A) 0.16
(B) 0.36
(C) 0.56
(D) 0.76
(E) 0.96

## Revisiting the Learning Goals

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- Trace the execution of and implement the value iteration algorithm for solving a Markov Decision Process.

