Constraint Satisfaction Problems: Backtracking Search

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Lecture 6

Based on work by K. Leyton-Brown, K. Larson, and P. van Beek

Outline

Learning Goals

By the end of the lecture, you should be able to

- Contrast depth-first search and backtracking search on a CSP.
- Trace the execution of the backtracking search algorithm.
- Trace the execution of the backtracking search algorithm with forward checking and/or arc consistency.
- Trace the execution of the backtracking search algorithm with forward checking and/or arc consistency and with heuristics for choosing variables and values.

Depth-first search v.s. Backtracking search on a CSP

Backtracking search is a special kind of Depth-first search, but they are not the same.

Why?

A CSP is commutative

- A CSP is commutative. Assigning values to variables in different orders will arrive at the same state.
- In each node, we should only consider one variable when generating successor states.

Backtracking Search

Algorithm 1 BACKTRACK(assignment, csp)

```
 1: if assignment is complete then return true
 2: var ← SELECT-UNASSIGNED-VARIABLE(csp)
 3: for all value in ORDER-DOMAIN-VALUES(var, assignment, csp) do
 4:     if adding {var = value} satisfies every constraint then
 5:         add {var = value} to assignment
 6:         result ← BACKTRACK(assignment, csp)
 7:         if result is true then return result
 8:     end if
 9:     remove {var = value} from assignment
10: end for
11: return false
```

Questions to consider

1. What inferences should we perform at each step of the search?
2. Which variable should we choose next?
   Which value of the variable should we try next?

Interleaving search and inferences

What inferences should be performed at each step in the search?

- Execute arc consistency algorithm before search.
- **Forward Checking**:
  - Simplified form of arc-consistency
  - Make every unassigned variable arc-consistent with the current variable.
- **Maintaining Arc Consistency (MAC)**:
  - Run the AC-3 algorithm

Backtracking with Inferences

Algorithm 2 BACKTRACK-INFERENCES(assignment, csp)

```
1: if assignment is complete then return true
2: var ← SELECT-UNASSIGNED-VARIABLE(csp)
3: for all value in ORDER-DOMAIN-VALUES(var, assignment, csp) do
4:     if adding {var = value} satisfies every constraint then
5:         add {var = value} to assignment
6:         inf-result ← INFERENCES(assignment, csp)
7:         if inf-result is true then
8:             add the inference results to assignment
9:             result ← BACKTRACK(assignment, csp)
10:            if result is true then return result
11:        end if
12:    end if
13:    remove {var = value} and the inference results from assignment
14: end for
15: return false
```

Backtracking Search with Forward Checking

Start with the initial assignment $x_0 = 0$.

Execute the backtracking search algorithm with forward checking.

CQ: Forward Checking

CQ: Consider the 4-queens problem with an empty assignment. Choose $x_0 = 0$. After this assignment, which of the following is the result of performing forward checking?

(A) $x_0 = 0, x_1 \in \{0, 1, 2, 3\}, x_2 \in \{0, 1, 2, 3\}, x_3 \in \{0, 1, 2, 3\}$

(B) $x_0 = 0, x_1 \in \{1, 2, 3\}, x_2 \in \{1, 2, 3\}, x_3 \in \{1, 2, 3\}$

(C) $x_0 = 0, x_1 \in \{2, 3\}, x_2 \in \{1, 3\}, x_3 \in \{1, 2\}$

(D) $x_0 = 0, x_1 \in \{3\}, x_2 \in \{1, 3\}, x_3 \in \{2\}$

CQ: Maintaining Arc Consistency

CQ: Consider the 4-queens problem with an empty assignment. Choose $x_0 = 0$. After this assignment, which of the following is the result of maintaining arc consistency?

(A) $x_0 = 0, x_1 \in \{2, 3\}, x_2 \in \{1, 3\}, x_3 \in \{1, 2\}$

(B) $x_0 = 0, x_1 \in \{3\}, x_2 \in \{1, 3\}, x_3 \in \{1\}$

(C) $x_0 = 0, x_1 \in \{3\}, x_2 \in \{1\}, x_3 \in \{1\}$

(D) $x_0 = 0, x_1 \in \{3\}, x_2 \in \{\}, x_3 \in \{1\}$

Which variable and value should we choose next?

Heuristics for selecting a variable

- **minimum-remaining-values** (MRV) heuristic: Choose the variable with the fewest values left in its domain.
- **degree** heuristic: Choose the variable that is involved in the largest number of constraints on other unassigned variables.
- When choosing a variable, apply the MRV heuristic first. Whenever there is a tie, use the degree heuristic to break ties.

Heuristics for selecting a value for a variable

- **least-constraining-value** heuristic: Select the value that **rules out the fewest values** (or **leaves the largest number of values**) for the neighbouring unassigned variables.

Backtracking with Inferences and Heuristics

Algorithm 3 BACKTRACK-INF-HEUR(assignment, csp)

```
1: if assignment is complete then return true
2: choose var based on MRV and DEGREE HEURISTICS
3: for all value in dom(var) chosen based on LCV HEURISTIC do
4:     if adding {var = value} satisfies every constraint then
5:         add {var = value} to assignment
6:         inf-result ← INFERENCES(assignment, csp)
7:         if inf-result is true then
8:             add the inference results to assignment
9:         end if
10:        result ← BACKTRACK(assignment, csp)
11:        if result is true then return result
12:    end if
13:    remove {var = value} and the inference results from assignment
14: end for
15: return false
```

CQ: Applying the least-constraining-value heuristic

CQ: Consider the following partial assignment for the 4-queens problem. (x_i denotes the row position of the queen in column i.)

$$x_0 = 0, x_1 \in \{2, 3\}, x_2 \in \{1, 3\}, x_3 \in \{1, 2\}$$

Based on the least-constraining-value heuristic,
which value of x_1 should we choose?

(A) $x_1 = 2$

(B) $x_1 = 3$

Revisiting the Learning Goals

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