Solution: SAMPLE SOLUTIONS

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1 Learning Goals

By the end of the exercise, you should be able to:

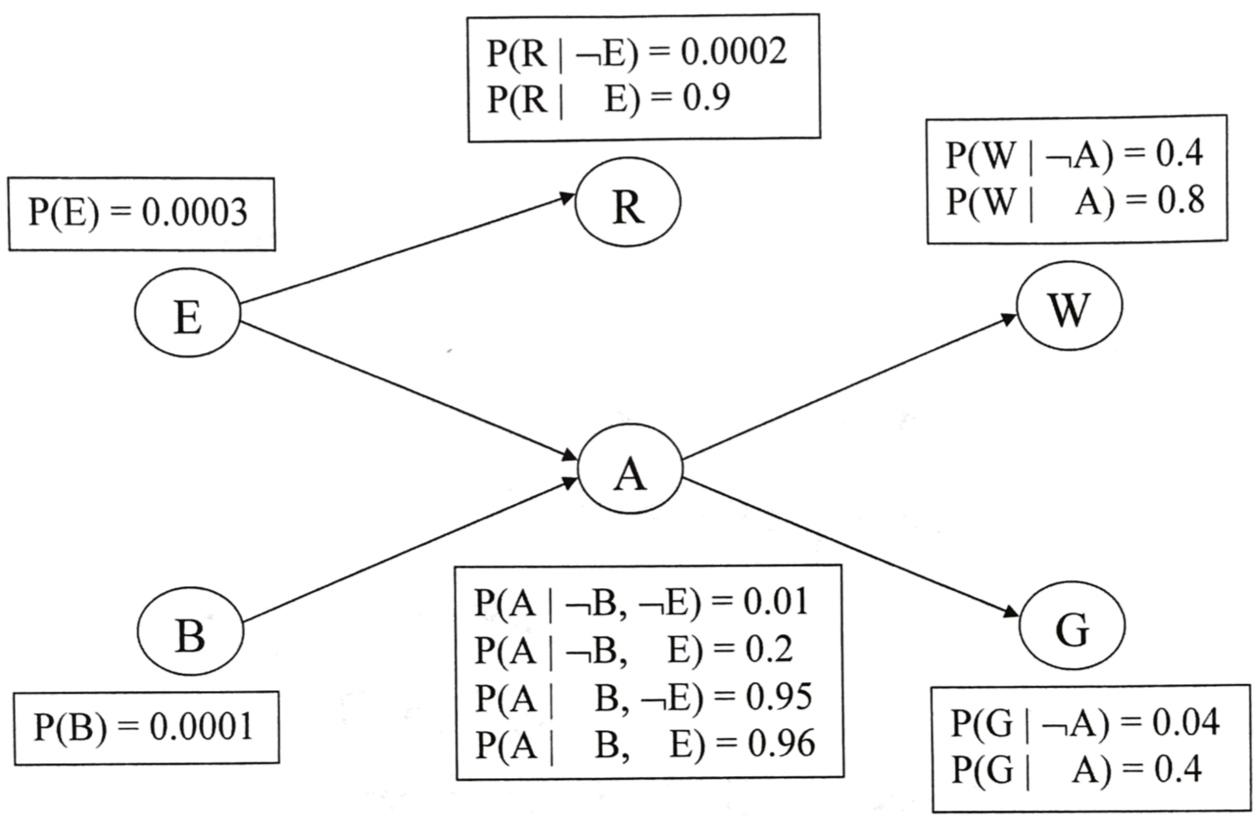
- Define factors. Manipulate factors using operations restrict, sum out, multiply and normalize.
- Compute a prior or a posterior probability given a Bayesian network.
- Describe/trace/implement the variable elimination algorithm for calculating a prior or a posterior probability given a Bayesian network.

2 The Holmes scenario

Mr. Holmes lives in a high crime area and therefore has installed a burglar alarm. He relies on his neighbors to phone him when they hear the alarm sound. Mr. Holmes has two neighbors, Dr. Watson and Mrs. Gibbon.

Unfortunately, his neighbors are not entirely reliable. Dr. Watson is known to be a tasteless practical joker and Mrs. Gibbon, while more reliable in general, has occasional drinking problems.

Mr. Holmes also knows from reading the instruction manual of his alarm system that the device is sensitive to earthquakes and can be triggered by one accidentally. He realizes that if an earthquake has occurred, it would surely be on the radio news.



3 Practice Questions

Question 1:

The following is a factor given in the table format.

f_1 :

X	Y	Z	Val
t	t	t	0.1
t	t	f	0.9
t	f	t	0.2
t	f	f	0.8
f	t	t	0.3
f	t	f	0.7
f	f	t	0.4
f	f	f	0.6

What is the result factor f_2 if we restrict factor f_1 to $X = t, Z = f$?
What is the result factor f_3 if we restrict factor f_1 to $X = t, Y = t, Z = f$?
Use the table format to represent them.

Solution: f_2 :

Y	Val
t	0.9
f	0.8

f_3 :

Val
0.9

Question 2:

Consider the factor given in the above question.

What is the result factor f_4 if we sum out X from factor f_1?

Solution: f_4 :

Y	Z	Val
t	t	0.4
t	f	1.6
f	t	0.6
f	f	1.4

Question 3:

The following is a factor given in the table format.

f_5 :

Z	Val
t	0.4
f	0.8

What is the result factor f_6 if we sum out Z from factor f_5?

Solution: f_6 :

Val
1.2

Questions 4:

The following are 2 factors given in the table format.

f_7 :

X	Y	Val
t	t	0.1
t	f	0.2
f	t	0.3
f	f	0.4

f_8 :

Z	Y	Val
t	t	0.4
t	f	0.3
f	t	0.2
f	f	0.1

What is the result factor f_9 if we multiply f_7 and f_8?

Solution: f_9 :

X	Y	Z	Val
t	t	t	0.04
t	t	f	0.02
t	f	t	0.06
t	f	f	0.02
f	t	t	0.12
f	t	f	0.06
f	f	t	0.12
f	f	f	0.04

Question 5:

The following are 2 factors given in the table format.

f_{10} :

X	Val
t	0.1
f	0.2

f_{11} :

Z	Y	Val
t	t	0.4
t	f	0.3
f	t	0.2
f	f	0.1

What is the result factor f_{12} if we multiply f_{10} and f_{11}?

Solution: f_{12} :

X	Y	Z	Val
t	t	t	0.04
t	t	f	0.02
t	f	t	0.03
t	f	f	0.01
f	t	t	0.08
f	t	f	0.04
f	f	t	0.06
f	f	f	0.02

Question 6:

Normalize the following factor f_{13} to produce f_{14}.

f_{13} :

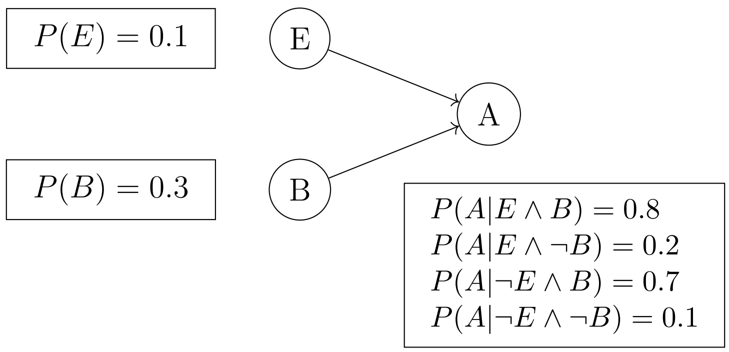
Z	Val
t	0.5
f	1.5

Solution: f_{14} :

Z	Val
t	0.25
f	0.75

Question 7:

Given the following Bayesian network (a portion of the network of the Holmes scenario), calculate $P(B|\neg A)$ using the variable elimination algorithm.



Solution: We will calculate the following conditional probability distribution using the variable elimination algorithm.

$$\begin{bmatrix} P(B|\neg A) \\ P(\neg B|\neg A) \end{bmatrix}$$

Step 1: Construct a factor for each conditional probability distribution.

Define factor $f_1(E)$ to correspond to $P(E = e)$:

E	Val
t	0.1
f	0.9

Define factor $f_2(B)$ to correspond to $P(B = b)$:

B	Val
t	0.3
f	0.7

Define factor $f_3(A, B, E)$ to correspond to $P(A = a|B = b, E = e)$:

A	B	E	Val
t	t	t	0.8
t	t	f	0.7
t	f	t	0.2
t	f	f	0.1
f	t	t	0.2
f	t	f	0.3
f	f	t	0.8
f	f	f	0.9

Step 2: Restrict the factors. Set the observed variables to their observed values.

Since we observe that A is false, we will restrict A in the original factors.

Define factor $f_4(B, E) = f_3(B, E, A = f)$:

B	E	Val
t	t	0.2
t	f	0.3
f	t	0.8
f	f	0.9

Step 3: Eliminate each hidden variable according to an order.

There is only 1 hidden variable E, we will sum it out.

There are 2 factors containing E, f_1 and f_4. We will mutliply these two factors together.

Define factor $f_5 = f_1 \times f_4$:

B	E	Val
t	t	0.02
t	f	0.27
f	t	0.08
f	f	0.81

Next, we will sum out E from the resulting factor.

Define factor $f_6 = \sum_E f_5(E)$:

B	Val
t	0.29
f	0.89

Step 4: Multiple the remaining factors.

Define factor $f_7 = f_2 \times f_6$:

B	Val
t	0.087
f	0.623

Step 5: Normalize the resulting factor.

f_8:

B	Val
t	0.1225
f	0.8775

Thus, $P(B|\neg A) = 0.1225$