Contents

1 Learning Goals

By the end of the exercise, you should be able to:

- Calculate prior, posterior, and joint probabilities using the sum rule, the product rule, the chain rule and Bayes' rule.
- Given a description of a domain or a probabilistic model for the domain, determine whether two variables are unconditionally independent.
- Given a description of a domain or a probabilistic model for the domain, determine whether two variables are conditionally independent given a third variable.

2 The Holmes scenario

Mr. Holmes lives in a high crime area and therefore has installed a burglar alarm. He relies on his neighbors to phone him when they hear the alarm sound. Mr. Holmes has two neighbors, Dr. Watson and Mrs. Gibbon.

Unfortunately, his neighbors are not entirely reliable. Dr. Watson is known to be a tasteless practical joker and Mrs. Gibbon, while more reliable in general, has occasional drinking problems.

Mr. Holmes also knows from reading the instruction manual of his alarm system that the device is sensitive to earthquakes and can be triggered by one accidentally. He realizes that if an earthquake has occurred, it would surely be on the radio news.

3 Practice Questions

3.1 Inferences using the joint distribution

Here is a joint distribution of the three random variables Alarm, Watson and Gibbon.

A			$\neg A$		
	G	$\neg G$		G	$\neg G$
W	0.032	0.048	W	0.036	0.324
$\neg W$	0.008	0.012	$\neg W$	0.054	0.486

Using the above information to calculate the following probabilities.

1. What is probability that **the alarm is NOT going and Dr. Watson is calling**?

2. What is probability that **the alarm is going and Mrs. Gibbon is NOT calling**?

3. What is the probability that **the alarm is NOT going**?

4. What is probability that **Dr. Watson is calling given that the alarm is NOT going**?

5. What is probability that **Mrs. Gibbon is NOT calling given that the alarm is going**?

3.2 Inferences using the prior and conditional probabilities

The prior probabilities:

$P(A) = 0.1$

The conditional probabilities

$P(W\|A) = 0.9$	$P(W\|\neg A) = 0.4$	$P(G\|A) = 0.3$	$P(G\|\neg A) = 0.1$
$P(W\|A \land G) = 0.9$	$P(W\|\neg A \land G) = 0.4$	$P(G\|A \land W) = 0.3$	$P(G\|\neg A \land W) = 0.1$
$P(W\|A \land \neg G) = 0.9$	$P(W\|\neg A \land \neg G) = 0.4$	$P(G\|A \land \neg W) = 0.3$	$P(G\|\neg A \land \neg W) = 0.1$

Using the above information to calculate the following probabilities:

1. What is probability that **the alarm is going, Dr. Watson is calling and Mrs. Gibbon is NOT calling**?

2. What is probability that **the alarm is NOT going, Dr. Watson is NOT calling and Mrs. Gibbon is NOT calling**?

3. What is the probability that **the alarm is NOT going given that Dr. Watson is calling**?

4. What is the probability that **the alarm is going given that Mrs. Gibbon is NOT calling**?

3.3 Unconditional and Conditional Independence

Question 1: Burglary, Alarm and Watson

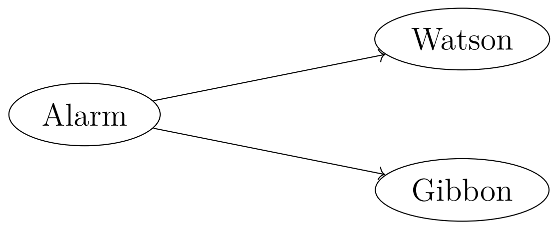
$P(B) = 0.1$ $\quad$ $P(A|B) = 0.9$ $\quad$ $P(A|\neg B) = 0.1$

$P(W|B \wedge A) = 0.8$ $\quad$ $P(W|\neg B \wedge A) = 0.8$ $\quad$ $P(W|B \wedge \neg A) = 0.4$ $\quad$ $P(W|\neg B \wedge \neg A) = 0.4$

1. Is Burglary independent of Watson?

2. Is Burglary conditionally independent of Watson given Alarm?

Question 2: Alarm, Watson, and Gibbon



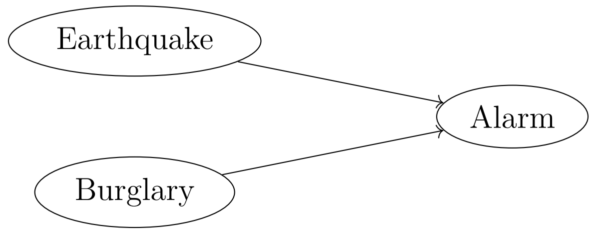
$P(A) = 0.1$ $\quad P(W|A) = 0.8$ $\quad P(W|\neg A) = 0.4$

$P(G|W \wedge A) = 0.4$ $\quad P(G|\neg W \wedge A) = 0.4$ $\quad P(G|W \wedge \neg A) = 0.1$ $\quad P(G|\neg W \wedge \neg A) = 0.1$

1. Is Watson independent of Gibbon?

2. Is Watson independent of Gibbon given Alarm?

Question 3: Earthquake, Burglary and Alarm



$P(E) = 0.1$ $\quad P(B|E) = 0.2$ $\quad P(B|\neg E) = 0.2$

$P(A|B \wedge E) = 0.9$ $\quad P(A|\neg B \wedge E) = 0.2$ $\quad P(A|B \wedge \neg E) = 0.8$ $\quad P(A|\neg B \wedge \neg E) = 0.1$

1. Is Earthquake independent of Burglary?

2. Is Earthquake conditionally independent of Burglary given Alarm?