Contents

1 Learning Goals

By the end of the exercise, you should be able to

- Trace the execution of the A* search algorithm using different heuristic functions.

2 Problem Description

An instance of the 8-puzzle consists of a 3×3 board with 8 numbered tiles and a blank space. Each tile has a number from 1 to 8. A tile adjacent to the blank space can slide into the space. The goal is to reach a specified goal state, such as the one shown below.

Initial State

5	3	
8	7	6
2	4	1

Goal State

1	2	3
4	5	6
7	8	

The 8-puzzle as a search problem:

State: Each state is given by $x_{00}x_{10}x_{20}, x_{01}x_{11}x_{21}, x_{02}x_{12}x_{22}$ where x_{ij} is the value in the space at column i and row j. If the space is blank, the value is zero. $i, j \in \{0, 1, 2\}$. $x_{ij} \in \{0, \ldots, 8\}$.

Initial state: $530, 876, 241$

Goal state: $123, 456, 780$

Action: Move the blank space up, down, left or right, wherever possible.

Successor function: The resulting state after taking one action.

Cost function: Each move has a cost of 1.

3 Instructions

You will trace the execution of the A* search algorithm with two different heuristic functions: The Manhattan distance heuristic and the Misplaced tiles heuristic.

Generate successors attempting to move the blank space in the following order: up, right, down, left.

When choosing which state to remove from the frontier, if there is a tie in the heuristic value, choose the state that comes earlier in lexicographical order (by treating each state as a 9-digit number).

4 A* Search with the Manhattan Distance Heuristic

For each state, the h value is the sum of the distances of the tiles from their goal positions (not counting the empty space as one tile). Each tile can only move horizontally or vertically. Thus, this is also called the Manhattan distance.

Note: The previous version of this notes was counting the empty square as one tile. This has been changed to be consistent with the textbook.

Complete the first five steps of the A* algorithm using the Manhattan distance heuristic.

Step 1: Add the initial state to the frontier.

5	3	
8	7	6
2	4	1

g = 0, h = 16, f = 0 + 16 = 16

frontier at the end of this step = $[((530, 876, 241), 16)]$

Step 2: Remove one state from the frontier. If the state is not a goal, add all of its successors to the frontier.

Solution: state removed:

5	3	
8	7	6
2	4	1

successors:

5	3	6
8	7	
2	4	1

5		3
8	7	6
2	4	1

frontier at the end of this step:
$[((503, 876, 241), 16), ((536, 870, 241), 18)]$

Step 3: Remove one state from the frontier. If the state is not a goal, add all of its successors to the frontier.

Solution: state removed:

5		3
8	7	6
2	4	1

successors:

5	3	
8	7	6
2	4	1

5	7	3
8		6
2	4	1

	5	3
8	7	6
2	4	1

frontier at the end of this step:
$[((053, 876, 241), 16), ((536, 870, 241), 18), ((530, 876, 241), 18), ((573, 806, 241), 18)]$

Step 4: Remove one state from the frontier. If the state is not a goal, add all of its successors to the frontier.

Solution: state removed:

	5	3
8	7	6
2	4	1

successors:

5		3
8	7	6
2	4	1

8	5	3
	7	6
2	4	1

frontier at the end of this step:
$[((536, 870, 241), 18), ((530, 876, 241), 18), ((573, 806, 241), 18),$
$((503, 876, 241), 18), ((853, 076, 241), 18)]$

Step 5: Remove one state from the frontier. If the state is not a goal, add all of its successors to the frontier.

Solution: All of the nodes in the frontier tie for the smallest heuristic value. We will break ties by choosing the state that is represented by the smallest 9-digit number.

state removed:

5	0	3
8	7	6
2	4	1

successors:

5	3	0
8	7	6
2	4	1

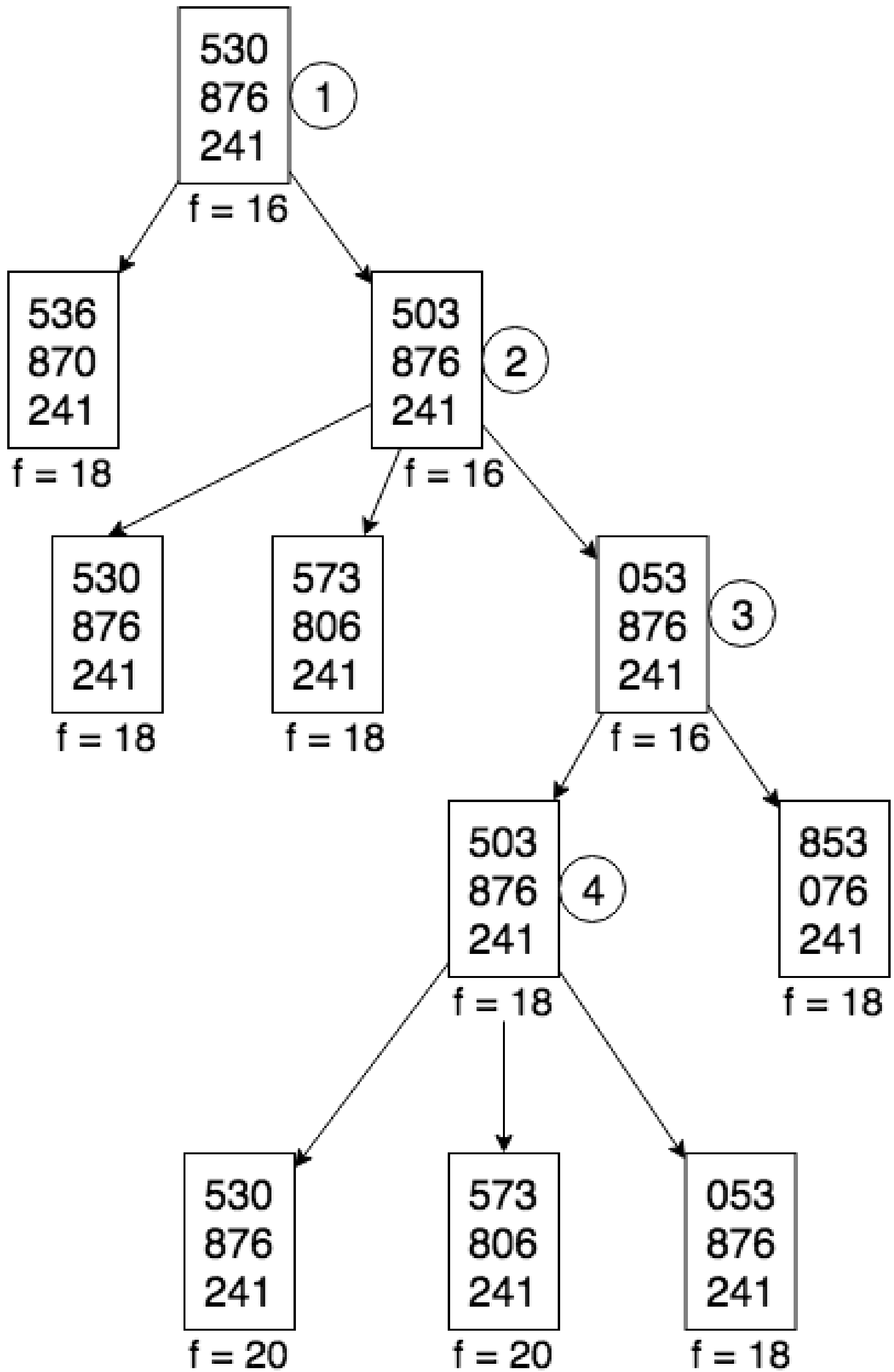
5	7	3
8	0	6
2	4	1

0	5	3
8	7	6
2	4	1

frontier at the end of this step:
$[((536, 870, 241), 18), ((530, 876, 241), 18), ((573, 806, 241), 18),$
$((853, 076, 241), 18), ((530, 876, 241), 20), ((573, 806, 241), 20), ((053, 876, 241), 18)]$

Solution:

The search tree of A* with the Manhattan Distance Heuristic



5 A* Search with the Misplaced Tile Heuristic

For each state, the h value is the number of tiles that are NOT in their goal positions (not counting the empty square as one tile).

Note: The previous version of this notes was counting the empty square as one tile. This has been changed to be consistent with the textbook.

Complete the first five steps of the A* algorithm using the Misplaced tile heuristic.

Step 1: Add the initial state to the frontier.

5	3	
8	7	6
2	4	1

g = 0, h = 7, f = 0 + 7 = 7

frontier at the end of this step: $[((530, 876, 241), 7)]$

Step 2: Remove one state from the frontier. If the state is not a goal, add all of its successors to the frontier.

Solution: successors:

5	3	6
8	7	
2	4	1

$f = 9$

5		3
8	7	6
2	4	1

$f = 7$

frontier at the end of this step $= [((536, 870, 241), 9), ((503, 876, 241), 7)]$

Step 3: Remove one state from the frontier. If the state is not a goal, add all of its successors to the frontier.

Solution: state removed:

5		3
8	7	6
2	4	1

successors:

5	3	
8	7	6
2	4	1

$f = 9$

5	7	3
8		6
2	4	1

$f = 8$

	5	3
8	7	6
2	4	1

$f = 8$

frontier at the end of this step:

$[((536, 870, 241), 9), ((530, 876, 241), 9), ((573, 806, 241), 8), ((053, 876, 241), 8)]$

Step 4: Remove one state from the frontier. If the state is not a goal, add all of its successors to the frontier.

Solution: There are two nodes with the smallest cost in the frontier. We will break ties using lexicographical order. In other words, interpret each state as a 9-digit number and remove the node with the smaller number.

state removed:

	5	3
8	7	6
2	4	1

successors:

5		3
8	7	6
2	4	1

$f = 9$

8	5	3
	7	6
2	4	1

$f = 9$

frontier at the end of this step:

$[((536, 870, 241), 9), ((530, 876, 241), 9), ((573, 806, 241), 8), ((503, 876, 241), 9), ((853, 076, 241), 9)]$

Step 5: Remove one state from the frontier. If the state is not a goal, add all of its successors to the frontier.

Solution: state removed:

5	7	3
8		6
2	4	1

successors:

5		3
8	7	6
2	4	1

$f = 9$

5	7	3
8	6	
2	4	1

$f = 10$

5	7	3
8	4	6
2		1

$f = 9$

5	7	3
	8	6
2	4	1

$f = 9$

frontier at the end of this step:

$[((536, 870, 241), 9), ((530, 876, 241), 9), ((503, 876, 241), 9), ((853, 076, 241), 9),$

$((573, 860, 241), 10), ((573, 846, 201), 9), ((573, 086, 241), 9)]$

Solution:

The search tree of A* with the Misplaced Tile Heuristic